Energy Saving Innovation, Vintage Capital, and the Green Transition*

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February 25, 2025

Abstract

We study a small open economy that must implement an emissions reduction plan and eventually phase out fossil fuel. R&D leads to the design of energy saving new machines. Endogenous scrapping eliminates old inefficient machines. We identify two distortions that delay the adoption and diffusion of energy saving technology: scrapping of old equipment and investment in new machines are both too low. The optimal policy to manage the energy transition thus combines a carbon tax with a profit tax to speed up exit, and an investment subsidy to speed up investment in new equipment. The optimal policy increases capital turnover, the diffusion of energy saving technology, and thereby mitigates the costs of the energy transition. Compared to a policy that exclusively relies on carbon taxes, the optimal policy could reduce the GDP loss of moving to net zero from 7.8 to 6.1% of GDP.

JEL classification: D21, D62, H23, O33, Q41, Q43.

Keywords: Energy saving innovation, vintage capital, emissions reduction.

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We are grateful to Philippe Aghion, Rick van der Ploeg, Mar Reguant and Matthias Rottner, and to seminar participants at the CEPR workshop on Sustainability and Public Policy in St. Gallen in January 2024.

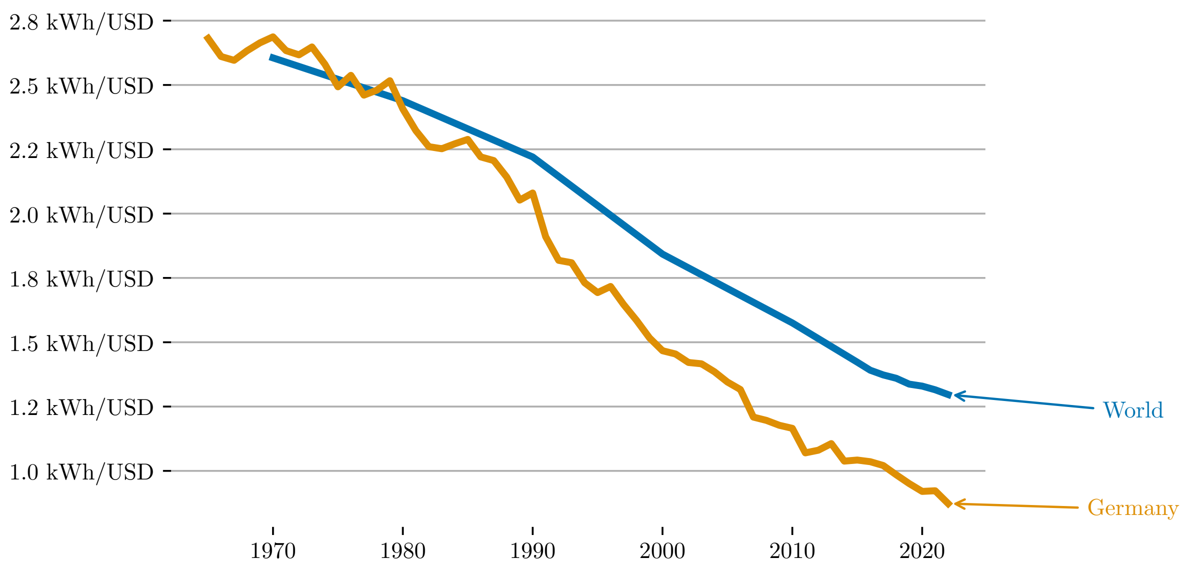
1 Introduction

To stop global warming, net emissions must go to zero. The European Union has committed to an emissions reduction plan to reach the net zero goal by 2050 (European Parliament, 2021). Since emissions are mostly related to fossil fuel, countries must phase out the use of coal, oil and gas, and replace them by renewables (water, solar, and wind) or other sources of clean energy. To achieve such goals, countries need to impose high carbon prices by levying a carbon tax or imposing tight quotas in an emissions trading scheme. However, by inflating energy prices, high carbon prices restrict economic performance and give rise to a negative climate economy trade-off. Governments should thus encourage environmentally friendly innovation to make the energy transition less costly.

Innovation may take several directions. Our focus is on energy saving innovation. Since current production predominantly relies on fossil fuel and the scaling up of green energy supply is bound to be slow, one must expect high energy prices when phasing out coal, oil and gas. Energy saving innovation reduces energy demand and, in turn, reduces the pressure on prices. An example is the oil crisis in the 1970s (Schiller, 1981). The jump in oil prices led to a recession. Thereafter, high oil prices boosted energy saving innovation and made advanced countries much less dependent on oil imports. To save on energy costs, consumers increasingly switched to more energy efficient cars. Oil prices declined. By the same logic, innovation and diffusion of energy saving technology should be an important element of managing the energy transition.

Indeed, reduced emissions growth in Germany in the last decade is to a large extent due to energy saving based on data from U.S. Energy Information Administration (2023); Energy Institute (2024); and Bolt and van Zanden (2024). Across the world, countries require less and less energy to produce a unit of GDP, see Figure 1. The size of these changes is substantial. The energy intensity in the world has declined from $2.6kWh$ in 1970 to less than $1.3kWh$ in 2022. In reducing energy intensity by 67%, Germany was even more successful in decoupling energy consumption and output growth. For any given supply of renewable energy, shrinking total energy demand necessarily implies a declining use of fossil fuel and thereby reduces the economy's carbon emissions. Notwithstanding other factors influencing de-carbonization, emissions per unit of energy declined significantly more in Germany compared to the worldwide average (Figure 2).

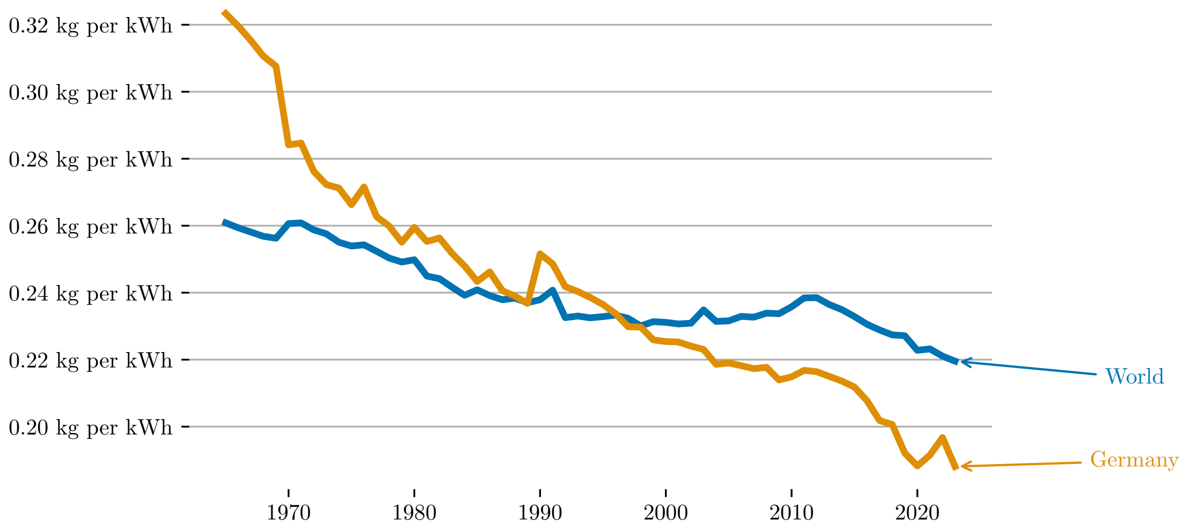
Figure 1: Energy Intensity of Output



Note: Energy intensity is measured as primary energy consumption per unit of gross domestic product, measured in kilowatt-hours per 2011$ (PPP).
Source: U.S. Energy Information Administration (2023); Energy Institute (2024); Bolt and van Zanden (2024), using Our World in Data.

Energy saving innovation can greatly facilitate emissions reductions and should thus be an important element of the energy transition to reach the net-zero-by-2050 goal. A key concern is how fast new technology can be implemented to reduce energy demand to prevent excessively rising energy prices in the early part of the transition. To highlight the role of innovation and the speed of technological diffusion, we construct a model of a small open economy with vintage capital and energy saving innovation. Energy demand draws on the supply of fossil fuel and renewable energy. Emissions are linked to the use of fossil fuel. To reduce carbon emissions, the government must impose a carbon tax high enough to phase out fossil fuel. Rising energy prices stimulate green energy supply, restrict energy demand, and lead to output losses. The production sector invests in energy saving R&D to reduce energy costs which makes high prices less damaging.

Figure 2: Emissions per Unit of Energy



Note: $CO2$ emissions per kWh of energy.
Source: Andrew and Peters (2024); U.S. Energy Information Administration (2023); Energy Institute (2024), using Our World in Data.

Different from existing literature, we assume that energy use is linked to the operation of machines. Each production unit consists of one machine and uses variable amounts of labor. Machines differ by their energy intensity, reflecting the state of technology at the date of implementation. The assumption that energy intensity is fixed over the entire life of a machine leads to a vintage capital model. The newest machines are the most efficient while old ones consume much more energy. To speed up the energy transition, it might become urgent to scrap old inefficient vintages and replace them with new and more energy efficient equipment. Firms decide when to discard old and inefficient machinery which makes the depreciation rate of capital endogenous. In essence, our framework introduces a specific model of creative destruction, with new technology replacing old and inefficient equipment. Importantly, investment of new machines and the scrapping of old ones both determine the speed of capital turnover and of technological diffusion.

We also allow for monopolistic competition and knowledge spillovers in private R&D and investment. We claim that mark-up pricing due to local market power from product differentiation is predominant in the production sector. The combination of imperfect competition and knowledge spillovers gives rise to specific frictions that impair and prolong the energy transition. As is standard in the innovation literature, external gains from knowledge spillovers reduce the rate of investment in new technology. A key novel result is that the presence of monopolistic rents makes firms hesitant to close down existing production units. Compared to a social optimum, both the rate of investment in new technology and the scrapping of old equipment are too low.

These frictions, in putting a brake on both ends of the creative destruction process, slow down the diffusion of new energy saving technology and contribute to a more unfavorable climate economy trade-off. The policy implications are that a carbon tax is not enough to manage the energy transition. Optimal policy includes (i) a carbon tax to phase out fossil fuel, (ii) an investment subsidy to internalize knowledge spillovers, and (iii) a profit tax to faster eliminate old, energy inefficient machines. The optimal policy increases capital turnover, speeds up the diffusion of energy saving technology, and reduces the output losses caused by the carbon tax. Our calibrated and estimated model indicates that using optimal investment and scrapping subsidies in addition to a carbon tax could reduce the economic loss of moving to net zero from 7.8 to 6.1% of GDP.

Our analysis goes beyond existing research in several dimensions. First, we link energy demand to the operation of machines, whereas most existing literature rationalizes energy demand by including energy in the production function with constant substitution between labor, capital and energy. We believe that this is a step to more explicit micro-foundations of energy use where reduced energy demand results from installing more energy efficient equipment. Second, the vintage capital model shows how energy savings is linked to the turnover of the capital stock by replacing old with more efficient new equipment. Third, the scrapping decision endogenizes both ends of the creative destruction cycle and determines, together with the rate of investment and R&D, the speed of technological diffusion of new technology. Fourth, we identify distortions in scrapping and R&D decisions that slow down the diffusion of energy saving innovation. These frictions arise from knowledge spillovers and rents to capital due to imperfect competition. Finally, our framework implies that carbon pricing might not be enough to optimally manage the energy transition, and calls for a richer menu of policies.

The next section reviews the literature and relates our results to existing research. Section 3 outlines the theoretical model and characterizes optimal policies. More technical parts are in the Appendix. Section 4 presents quantitative results. Section 5 concludes.

2 A Review of Existing Literature

According to (Stern, 2008), greenhouse gas emissions are the largest market failure in the world. The pioneering work of Nordhaus and Boyer (2000) inspired the development of integrated assessment models (IAMs) to analyze the mutual interaction of climate change and economic growth, such as Golosov et al. (2014). Carbon emissions from economic activity give rise to climate change which feeds back negatively due to damages from global warming. To stop the vicious cycle, the world must decarbonize and move to net zero very fast. Our analysis focuses on a small open economy that cannot affect global warming but nevertheless is bound by international agreements to manage a transition to net zero. We thus take climate change as an exogenous event and simply ask how the country could manage an energy transition that minimizes economic cost.

With this perspective, we borrow from existing integrated assessment models such as Golosov et al. (2014) and modify the model in three novel ways. First, instead of introducing energy as an additional input in the production function, we link energy demand to the use of capital, with energy intensity of machines differing by vintages. Energy savings thus results not from a mechanical substitution of energy by other factors but from explicit R&D effort of firms to design more energy efficient machines. Second, we introduce vintage capital. Once a machine is in place, energy intensity is fixed over its entire life. Average energy intensity thus changes only slowly as old machines get replaced by more energy efficient new ones. The speed of diffusion of energy saving technology thus depends on the rate of new investment and the scrapping of old capital. Specifically, we allow for scrapping which endogenizes the depreciation rate of capital. A higher scrapping rate means that old equipment is replaced earlier with more efficient new machines, thereby speeding up the diffusion of new technology. Finally, we focus on a small, open economy that must manage a green transition subject to an exogenously specified emission reduction plan that follows from binding international agreements. We believe that this assumption makes our model practically relevant for many smaller countries. We derive optimal policies to comply with the emission constraint.

Existing research emphasizes the need for optimal carbon pricing (Nordhaus, 1991) and R&D support to manage a transition to net zero emissions (Blanchard et al., 2023). The range of estimates for optimal carbon prices is substantial, reflecting different assumptions on climate damages and social discount rates. Early on, Nordhaus (1991) suggested that the optimal carbon tax should be around 30 dollars per ton of CO2 while Stern (2008) argued for 250 dollars. The difference is mainly due to the higher discount rate of 1.5% used by Nordhaus (1991), compared to 0.1% by Stern (2008). In Golosov et al. (2014), optimal carbon prices are higher, equal to around 57 dollars per ton of CO2 when using a discount rate similar to Nordhaus (1991), and 500 dollars when using the rate in Stern (2008). The main reason is that these authors estimate higher damages from emissions and much slower depreciation of carbon stocks. Folini et al. (2024) show that simply adjusting the damage function from linear to quadratic results in a four times higher optimal carbon tax. Barrage (2018) puts more weight to future generations and applies a social discount rate higher than that of private households. Using the methodology of Golosov et al. (2014), she then arrives at an estimate of 183 dollars per ton of CO2 instead of 57 dollars. Barrage (2020) accounts for the presence of distorting taxes on capital and labor income and finds that optimal carbon taxes are 8-24% lower compared to Pigovian levels.

Our analysis, however, isn't directly related to this literature. In a small open economy, climate damages and global warming are disconnected from own activity and are taken as exogenous events. A single country is simply too small to have any noticeable climate impact, but nevertheless must comply with international agreements to stop global warming. From a national perspective, policy is concerned to fulfill the commitment to an emissions reduction plan with the smallest cost to the national economy. The government must thus find an optimal trade-off in using carbon pricing and innovation policies to satisfy the national climate plan. In our framework, optimal carbon pricing is determined by the exogenously imposed emissions constraint.

Technological progress plays a pivotal role in reducing emissions and mitigating temperature increases. By enhancing energy efficiency and reducing the reliance on carbon-intensive energy sources, innovation reduces the need for high carbon taxes to achieve the climate goals (Coppens et al., 2024). A seminal study on the role of innovation is Acemoglu et al. (2012) who distinguish between green and dirty production. Dirty production causes emissions and must be restricted. Knowledge spillovers in sectoral innovation, however, introduce a path dependency that is difficult to overcome. Since green technology is young, there is little knowledge to build on when doing R&D. The dirty sector, in contrast, can draw on a large knowledge stock which favors even more dirty innovation. To shift innovation from dirty to green technology thus requires high carbon taxes and possibly specific subsidies to green innovation early on. Delaying such policies risks entrenching reliance on dirty technology, making the transition eventually more challenging and expensive (Acemoglu et al., 2016). Our model, in contrast, features only one manufacturing sector where the operation of machines requires energy that stems from the use of fossil fuel and clean renewable sources. Our focus is on energy-saving innovation which reduces energy demand in manufacturing and, for any given supply of renewable energy, reduces the need for fossil fuel. Increasing carbon taxes inflates energy prices, thereby stimulating the supply of green energy and inducing energy saving innovation in manufacturing.

A large body of existing research studies innovation in a Schumpeterian framework of creative destruction, see Aghion et al. (2014) for a comprehensive summary. Our approach is distinct in studying innovation in a vintage capital model that endogenizes both ends of creative destruction. Technology is embodied in new machines, remains fixed for the remaining life-time, and can change only when old machines are replaced by new ones. Diffusion of new technology is slow, depending on endogenous investment and scrapping rates. When new technology becomes available, production takes place in a heterogeneity of plants using old, inefficient and new, efficient capital. In a sense, this mechanism creates 'stranded assets' similar to Jin et al. (2024) and van der Ploeg and Rezai (2020) that are endogenously liquidated (scrapped) when old technology becomes less profitable. Investment and scrapping are mutually dependent. Empirically, Brueckner et al. (2024) examined how the replacement of old capital by new investment could be accelerated by policy. They analyze the benefits of a Cash-for-Clunkers program in the aviation sector in the US and demonstrate that this program can yield net social benefits compared to merely selling older planes upon retirement.

In the same spirit, Rozenberg et al. (2020) show that optimal carbon taxes might minimize the costs of the green transition but can also lead to stranded assets with under-utilization of existing equipment. Stranded assets appear as a negative unintended consequence of carbon taxes. A similar trade-off between a faster transition and more asset write-offs is also present in our model. Unlike Rozenberg et al. (2020), our model combines monopolistic competition and endogenous depreciation (scrapping) which leads to different policy implications. We find that firms tend to use existing technology too long to enjoy a continuing stream of monopolistic profits. Consequently, early retirement of old capital is desirable to create room for new establishments with better technology.

Although the context differs, the argument is in the spirit of Aghion et al. (2019) who study the implications of credit constraints for innovation. They argue that '...better access to credit makes it easier for entrepreneurs to innovate; on the other hand, better credit access allows less efficient incumbent firms to remain longer on the market, thereby discouraging entry of new and potentially more efficient innovators.' Using French data, they find that these countervailing effects result in an inverted-U relationship between credit access and productivity growth.[1] In our model, these forces result in a socially

[1]In the same vein, Gutierrez and Philippon (2019) document that the elasticity of entry with respect

optimal scrapping rate that exceeds the private one. The optimal policy combines a carbon tax with a profit tax and an investment subsidy. The tax cum subsidy policy facilitates exit and at the same time encourage investment in new plants, thereby speeding up the diffusion of energy saving technology.[2]

The typical modeling of energy demand includes energy as an additional input in the production function. With Cobb Douglas and CES technology, the elasticity of substitution is constant over time. Depending on relative prices, inputs can be smoothly substituted for each other in the short- and the long-run. In contrast, our model links energy demand to the use of capital. Demand is thus fixed in the short-run and elastic in the long-run. It changes along with capital accumulation and increasing use of energy saving technology. We believe this approach to be not only intuitive but also in line with empirical research. For example, the meta-analysis of the price elasticity of energy demand conducted by Labandeira et al. (2017) suggests that short-term demand is quite inelastic, with elasticities ranging from -0.017 for heating oil to -0.293 for gasoline. Long-term elasticities are substantially larger, with estimates ranging from -0.185 for heating oil to -0.773 for gasoline. These results suggest, in line with our modeling, that energy demand is relatively unresponsive to price changes in the short term but becomes more elastic in the long term.

3 Theoretical Model

This section presents a vintage capital model with energy saving innovation. The use of capital requires energy which stems from renewable and fossil fuel.

to Tobin's Q has declined since the late 1990s, attributed not to technological costs or returns to scale, but to increased lobbying of incumbents and regulation.

[2] In Barrage (2020), the optimal carbon tax should be lower in the presence of other distorting taxes. Our analysis considers market distortions on top of climate externalities. Policy should thus combine carbon taxation with additional corrective taxes and subsidies.

3.1 Production

Assembling final goods requires differentiated inputs y_t^i from a set of varieties $i \in K_t$, using technology $Y_t = K_t^{1/(1-\sigma)} \left[\int_0^{K_t} \left(y_t^i\right)^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}$. The elasticity of substitution is constant, $\sigma > 1$. The term $K_t^{1/(1-\sigma)}$ shuts off gains from specialization. Instead, we assume knowledge spillovers from the design and introduction of new inputs. Accumulated knowhow determines plant-level productivity z_t as below. Given prices p_t^i, cost minimization leads to demand for input i,

$$y_t^i = \left(P_t/p_t^i\right)^{\sigma} Y_t/K_t, \tag{1}$$

where $P_t = \left[\frac{1}{K_t} \int_0^{K_t} \left(p_t^i\right)^{1-\sigma} di \right]^{1/(1-\sigma)}$ is the input price index, and $\int_0^{K_t} p_t^i y_t^i di = P_t Y_t$ is total cost.[3] The homogeneous final good is the *numeraire*. Perfect competition among assembling firms determines prices by the break-even condition, $Y_t - P_t Y_t = 0$. In symmetric equilibrium, input prices are

$$p_t = P_t = 1. \tag{2}$$

Inputs are produced in K_t specialized plants. A plant uses 1 unit of capital (a machine) and hires l_t^i workers to produce a differentiated input y_t^i. Plants are local monopolists but must compete with many close substitutes. Demand for own output $y_t^i = D_t / \left(p_t^i\right)^{\sigma}$ in (1) is downward sloping. Aggregate demand $D_t = P_t^{\sigma} Y_t / K_t$ is taken as given. The own price elasticity of demand is $\sigma = -\frac{p_t^i}{y_t^i} \frac{dy_t^i}{dp_t^i}$. All plants operate the same technology, face the same elasticity σ, and pay the same wage w_t. Decisions are thus symmetric,

$$\pi_t = \max_{l_t} \; p_t y_t - w_t l_t \quad s.t. \quad y_t = y\left(l_t, z_t\right) = z_t l_t^{1-\alpha}. \tag{3}$$

Using $y_{l,t} \equiv \frac{dy_t}{dl_t}$, the first order condition is $\left(p_t + y_t \frac{dp_t}{dy_t}\right) y_{l,t} = w_t$, or

$$p_t y_{l,t} = \rho \cdot w_t, \quad \pi_t = \bar{\pi} \cdot \alpha p_t y_t. \tag{4}$$

[3] Detailed solutions are documented in a separate technical appendix which is available upon request.

We conveniently define $\rho \equiv \frac{\sigma}{\sigma - 1} \geq 1$ and $\bar{\pi} \equiv 1 + \frac{\rho - 1}{\rho}\frac{1-\alpha}{\alpha} \geq 1$.[4] Plants restrict employment and output to exploit market power. Marginal revenue from expanding employment is thus a mark-up ρ over the competitive wage rate. Profit consists of two components, a competitive return on capital $\alpha p_t y_t$ and a monopolistic rent $\frac{\rho - 1}{\rho}(1-\alpha) p_t y_t$. If market power were absent ($\sigma \to \infty$, $\rho \to 1$ and $\bar{\pi} \to 1$), the plant would earn a profit no more than the competitive return on capital, $\pi_t = \alpha p_t y_t$.

A manufacturer operates many plants, N_{t-1} of them, each producing a unique input. She invests I_t of the final good to create I_t new product lines in I_t plants, each with one design and one unit of capital (a machine). Equipment investment and R&D are one process. Capital and varieties thus grow by

$$N_t = I_t + \phi_t N_{t-1}, \quad z_t = \bar{z}\left(N_{t-1}\right)^{\omega}, \quad \omega \geq 0. \tag{5}$$

Each firm and all its plants take productivity z_t as given. In equilibrium, productivity rests on cumulative knowledge N_{t-1} that is created from past investment in new designs. Absent spillovers ($\omega = 0$), productivity would be a constant $\bar{z}$.

Manufacturers allocate management effort m_t to each of its establishments which raises the success probability. A plant survives with probability $\phi_t = \phi(m_t)$ and fails with probability $1-\phi_t$. In other words, bad management increases business failure. After risk is resolved, the manufacturer continues with $K_t = \phi_t N_{t-1}$ plants, each producing a quantity y_t of the specialized input and earning profit π_t. Plants are identical, except for the energy efficiency of its machinery which differs across vintages. Operating a vintage j machine, installed at date $t-j$, consumes energy ϵ_{t-j} at a cost $\epsilon_{t-j} p_t^E$, where p_t^E is the energy price. The energy intensity ϵ_{t-j} depends on technology at the date of installation and is constant thereafter. The plant earns net of tax profit $(1-t_t^y)\,\pi_t$ with probability ϕ_t, and nothing if it is closed down. Manufacturers also receive an ex post investment subsidy s_t per plant.[5] We assume the success probability $\phi_t = \phi(m_t)$ to be linearly increasing in

[4] Multiply the first order condition by l_t and use $l_t y_{l,t} = (1-\alpha)\, y_t$ to get $(1-\alpha)\, p_t y_t = \rho w_t l_t$. Using this to eliminate wage costs gives $\pi_t = \frac{\alpha + \rho - 1}{\rho} p_t y_t$ which leads to the second equation.

[5] Investment could receive an upfront subsidy on I_t, or an ex post subsidy s_t. The second approach

management effort m_t while managerial cost $b_t = b(m_t)$ is convex increasing. Expected profit of a vintage j plant in period t is

$$\pi^e_{t-j,t} = (1 - t^y_t)\,\pi_t \phi(m_t) + s_t - \epsilon_{t-j} p^E_t - b(m_t)\,. \tag{6}$$

At date $t - j$, the firm installed I_{t-j} machines (use $I_{t,t} \equiv I_t$). Only a fraction ϕ_t of old vintages that existed at the end of $t-1$, survives to the end of t: $I_{t-j,t} = \phi_t I_{t-j,t-1}$. Given symmetry in ϕ_t, the range of varieties accumulates as in (5). However, new machines are endowed with the newest technology ϵ_t, fixing energy needs for the rest of life. Vintage j was equipped at date $t-j$ with technology ϵ_{t-j}. Average energy intensity ϵ^a_t is a weighted average of new and old vintages, $\epsilon^a_t \equiv \sum_{j\geq 0} \epsilon_{t-j} \cdot \frac{I_{t-j,t}}{N_t}$. Since $N_t = \sum_{j\geq 0} I_{t-j,t}$, weights add up to one. Writing out the sum and using $I_{t-j,t} = \phi_t I_{t-j,t-1}$ gives $\epsilon^a_t = \frac{\epsilon_t I_{t,t} + \phi_t\left[\epsilon_{t-1} I_{t-1,t-1} + \epsilon_{t-2} I_{t-2,t-1} + \ldots\right]}{N_t}$. By definition, the square bracket in the numerator is $\epsilon^a_{t-1} N_{t-1}$. Average energy intensity thus changes by

$$\epsilon^a_t = \epsilon_t \cdot \frac{I_t}{N_t} + \epsilon^a_{t-1} \cdot \frac{\phi_t N_{t-1}}{N_t}, \qquad E_t = \epsilon^a_{t-1} N_{t-1}. \tag{7}$$

Total energy demand E_t depends on average energy intensity ϵ^a_{t-1} across all vintages $t-1$ and older. New machines start producing in $t+1$. The energy intensity of capital ϵ^a_t next period reflects energy saving technology embodied in new and old machines, with weights adding up to one, $\frac{I_t}{N_t} + \frac{\phi_t N_{t-1}}{N_t} = 1$. Average energy intensity is much slower to adjust than energy efficiency of new machines.[6] In a steady state, however, investment merely replaces depreciated capital, $I = (1-\phi)\,N$, and energy intensity is the same across new and old vintages, $\epsilon = \epsilon^a$.

The newest plant producing in t was built last period. By symmetry, summing up profits results in $\pi^e_t N_{t-1} \equiv \sum_{j\geq 1} \pi^e_{t-j,t} I_{t-j,t-1} = \left[(1 - t^y_t)\,\pi_t \phi_t + s_t - \epsilon^a_{t-1} p^E_t - b_t\right] N_{t-1}$.

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is often recommended in tax theory (see the review in Keuschnigg (2011)).

[6]For later use, note derivatives $\frac{d\epsilon^a_t}{dI_t} = -\frac{\epsilon^a_t - \epsilon_t}{N_t}$, $\frac{d\epsilon^a_t}{dN_{t-1}} = \phi_t \frac{\epsilon^a_{t-1} - \epsilon^a_t}{N_t}$ and so forth.

The manufacturer thus collects an average profit per plant equal to

$$\pi_t^e = (1 - t_t^y)\,\pi_t \phi_t + s_t - \epsilon_{t-1}^a p_t^E - b_t. \tag{8}$$

Managers are offered $w_t^m = b(m_t)\,N_{t-1}$ for managing N_{t-1} plants, to compensate for effort cost (see the household section below).

3.2 R&D and Investment

The capital stock is related to the number of plants which changes by $N_t = I_t + \phi_t N_{t-1}$. Investment is subject to adjustment costs $\psi_t = \psi(I_t, N_{t-1}) = \frac{1}{2}\bar{\psi} N_{t-1} (I_t/N_{t-1} - (1 - \phi_t))^2$, as in 'Q-theory'. Adjustment costs make firms stretch investment over time and vanish in a steady state. Manufacturers also invest R_t in energy saving R&D. Cumulative R&D increases knowledge in designing more energy efficient machines,[7]

$$A_t = \varphi(R_t) + \delta A_{t-1}, \quad \epsilon_t = \epsilon(A_{t-1}). \tag{9}$$

R&D creates knowledge on energy saving solutions with concave technology, $\varphi' > 0 > \varphi''$. We specify $\varphi = \bar{\varphi}\left(R + \bar{R}\right)^\gamma$ with $\gamma < 1$. More knowhow A_{t-1}, in turn, reduces the energy intensity of new machines, $\epsilon' < 0 < \epsilon''$. We specify $\epsilon_t = \bar{e} \cdot e^{-\nu A_t}$.

Dividend payouts of manufacturers are cash-flow minus retained earnings which must pay for investment I_t in new plants, installation costs ψ_t, and R&D spending R_t,

$$\chi_t = \pi_t^e N_{t-1} - I_t - \psi_t - R_t. \tag{10}$$

Firm value is $V_t = \chi_t + V_{t+1}/(1 + i_t)$. The value function depends on three predetermined states, N_{t-1}, A_{t-1}, and ϵ_{t-1}^a, with values $\lambda_t^N \equiv \frac{dV_t}{dN_{t-1}}$ and $\lambda_t^A \equiv \frac{dV_t}{dA_{t-1}}$. A higher energy intensity ϵ_{t-1}^a inflates costs and reduces value, $\frac{dV_t}{d\epsilon_{t-1}^a} < 0$. In defining $\lambda_t^E \equiv -\frac{dV_t}{d\epsilon_{t-1}^a} > 0$, we interpret λ_t^E as measuring the returns to energy saving innovation. The Bellmann problem

[7] For lean modeling, we exclude knowledge spillovers that would result in a positive R&D externality. Setting a carbon tax inflates energy prices and, thereby, already sets an incentive for cost reducing R&D.

is $V\left(N_{t-1}, A_{t-1}, \epsilon^a_{t-1}\right) = \max_{I_t, R_t, m_t} \chi_t + V\left(N_t, A_t, \epsilon^a_t\right) / \left(1 + i_t\right)$ subject to the constraints stated above. Noting the effects on average energy intensity in (7) eventually gives

$$
\begin{aligned}
I_t &: 1 + \psi_{I_t} = \frac{\lambda^N_{t+1}}{1 + i_t} + \frac{\epsilon^a_t - \epsilon_t}{N_t} \frac{\lambda^E_{t+1}}{1 + i_t}, \quad R_t : 1 = \varphi'\left(R_t\right) \frac{\lambda^A_{t+1}}{1 + i_t}, \\
m_t &: b'\left(m_t\right) = \phi'\left(m_t\right) \left[\left(1 - t^y_t\right) \pi_t + \frac{\lambda^N_{t+1}}{1 + i_t} - \frac{\epsilon^a_{t-1} - \epsilon^a_t}{N_t} \frac{\lambda^E_{t+1}}{1 + i_t}\right].
\end{aligned} \tag{11}
$$

Envelope conditions on capital, average energy efficiency, and knowledge are

$$
\begin{aligned}
N_{t-1} &: \lambda^N_t = \pi^e_t - \psi_{N_{t-1}} + \phi_t \left[\frac{\lambda^N_{t+1}}{1 + i_t} - \frac{\epsilon^a_{t-1} - \epsilon^a_t}{N_t} \frac{\lambda^E_{t+1}}{1 + i_t}\right], \\
A_{t-1} &: \lambda^A_t = \delta \frac{\lambda^A_{t+1}}{1 + i_t} - \epsilon'\left(A_{t-1}\right) \frac{I_t}{N_t} \frac{\lambda^E_{t+1}}{1 + i_t}, \\
\epsilon^a_{t-1} &: \lambda^E_t = p^E_t N_{t-1} + \phi_t \frac{N_{t-1}}{N_t} \frac{\lambda^E_{t+1}}{1 + i_t}.
\end{aligned} \tag{12}
$$

The firm's decisions are partly driven by the need to save energy when prices are high. By improving the average energy efficiency ϵ^a_t of the capital stock, a manufacturer can reduce future energy consumption $E_{t+1} = \epsilon^a_t N_t$ as in (7). The value λ^E_{t+1} of future cost savings resulting from a marginal increase in ϵ^a_t follows from the forward solution of (12.iii). The need to save energy motivates R&D to accumulate more knowledge in energy saving solutions. The envelope condition (12.ii) indicates the marginal value λ^A_t of knowledge. Accumulating knowhow facilitates a more energy efficient design of new machines ($\epsilon' < 0$). When firms expect rising energy costs, the value λ^E_{t+1} of future cost savings from improving energy efficiency increases and makes energy saving knowhow more valuable (λ^A_{t+1} rises). Accordingly, by (11.ii), firms invest more in R&D.

The value λ^N_t of a new plant reflects future returns in two dimensions: (i) the present value of future profits from introducing a new product line (captured by λ^N_{t+1}); and (ii) the value of energy savings from faster diffusion of new technology (captured by λ^E_{t+1}). Replacing old energy intensive machines by more efficient new ones reduces energy demand per unit of capital, $\epsilon_t < \epsilon^a_t$ and $\epsilon^a_t < \epsilon^a_{t-1}$, and thereby boosts energy efficiency. When new machines are more energy efficient, $\epsilon_t < \epsilon^a_t$, they become more valuable as captured by

$(\epsilon_t^a - \epsilon_t)\,\lambda_{t+1}^E$ in (11.i) which strengthens incentives to invest, beyond the present value of marginal profits as captured by λ_{t+1}^N. By (7), a higher rate of investment, in turn, speeds up the diffusion of new energy saving technology. In the long-run, old and new machines are identical ($\epsilon = \epsilon^a$), making the energy saving motive of investment disappear. Substituting for $\psi_{I,t}$ in (11.i), one can express investment $I_t = \left[1 - \phi_t + \frac{1}{\psi}\left(Q_t^N - 1\right)\right] N_{t-1}$ in terms of Tobin's Q, $Q_t^N \equiv \frac{\lambda_{t+1}^N}{1+i_t} + \frac{\epsilon_t^a - \epsilon_t}{N_t}\frac{\lambda_{t+1}^E}{1+i_t}$, which is augmented to account for the (better) energy efficiency of new equipment.

Finally, investing in management makes firms more successful in avoiding failure and in continuing profitable business. Managerial investment is optimal if marginal cost $b'(m_t)$ is balanced by marginal gains of continuation, see (11.iii). These gains consist of the present value of profits $(1 - t_t^y)\,\pi_t + \frac{\lambda_{t+1}^N}{1+i_t}$ per establishment that is kept alive, net of the increase in future energy costs from continuing less efficient equipment. Continuing old and less efficient plants ($\epsilon_{t-1}^a > \epsilon_t^a$) slows down the diffusion of energy saving technology and raises future costs by $\frac{\epsilon_{t-1}^a - \epsilon_t^a}{N_t}\frac{\lambda_{t+1}^E}{1+i_t}$ which reduces the incentive to invest in management. To speed up the implementation of new technology, firms step up investment in new equipment (see above) and, at the same time, scrap old, inefficient equipment more rapidly. They do so by choosing a somewhat lower management intensity and accepting a lower survival rate. This energy saving motive of investment and scrapping is temporary, vanishes in the long-run when new and old machines are equally efficient, but speeds up the diffusion of new technology in the transition.

Government policy importantly affects investment, scrapping and energy saving R&D. An ex post investment subsidy s_t raises the manufacturer's expected profit π_t^e and, by (12.i), the value λ_t^N of a plant. For any given value of the profit tax, the subsidy boosts investment in new plants. If it is generous enough, it will more than compensate for the negative effect of the profit tax t_t^y to speed up business creation and the implementation of new technology. The profit tax, in turn, reduces the marginal value of continuing an old plant in (11.iii) and thereby stimulates scrapping of old and energy inefficient equipment. Both policy interventions together speed up the diffusion of new energy saving technology in the transition. Finally, imposing a substantial carbon tax triggers an increase in energy prices p_t^E which is needed to phase out fossil fuel. On top of that, high energy prices lead manufacturers not only to invest more in energy saving R&D, but also to scrap old machines and replace them with new equipment at a faster rate, thereby speeding up the diffusion of new technology.

3.3 The Energy Market

We aggregate the different sources of energy into two broad categories which are assumed to be perfect substitutes in demand and record the same price p_t^E. Currently, renewable energy such as solar, wind and water power is far from sufficient to cover energy demand. The major part stems from fossil fuel (coal, oil and gas). The energy transition must shift supply from fossil fuel to renewables and other clean sources which may happen in two ways: (i) reducing energy demand relative to a given supply of renewables cuts residual demand for fossil fuel; (ii) expanding the green sector further crowds out the use of fossil energy. Both effects reduce carbon emissions.

To focus on energy demand and the role of energy savings technology, we deliberately keep the energy sector simple. Firms must invest final goods to build solar panels, wind mills and large water power stations that generate renewable energy. Scaling up green energy X_t leads to progressively increasing costs, $J(X_t) = \bar{x}^{-1/\mu} X_t^{1+1/\mu} / (1+1/\mu)$.[8] Profit maximization determines supply by

$$\chi_t^X = \max_{X_t} \; p_t^E X_t - J(X_t) \quad \Rightarrow \quad p_t^E = J'(X_t), \quad X_t = \bar{x} \left(p_t^E\right)^{\mu}. \tag{13}$$

Increasing output raises marginal cost and thus requires higher energy prices. Using the functional form results in a supply function with price elasticity μ.

Producing fossil fuel in quantity F_t requires to invest ξF_t of final goods for 'extraction'. Producers use a linear technology with constant unit cost ξ. The assumption may be

[8] Final output embodies capital and labor. We thus implicitly assume that the factor content of production is the same in all sectors.

justified by the fact that the supply of fossil fuel reserves such as coal and maybe also oil and gas are infinitely large compared to what can be used to stop global warming. The use of fossil fuel leads to carbon emissions θF_t where θ is a constant emissions coefficient. The government levies a carbon tax τ_t per unit of carbon, creating a tax liability $\tau_t\theta$ per unit of fossil fuel. The purpose is to reduce emissions by raising production cost relative to green energy and thereby phase out the supply of fossil fuel. With linear cost, supply is infinitely elastic and is determined in equilibrium by a zero profit condition,

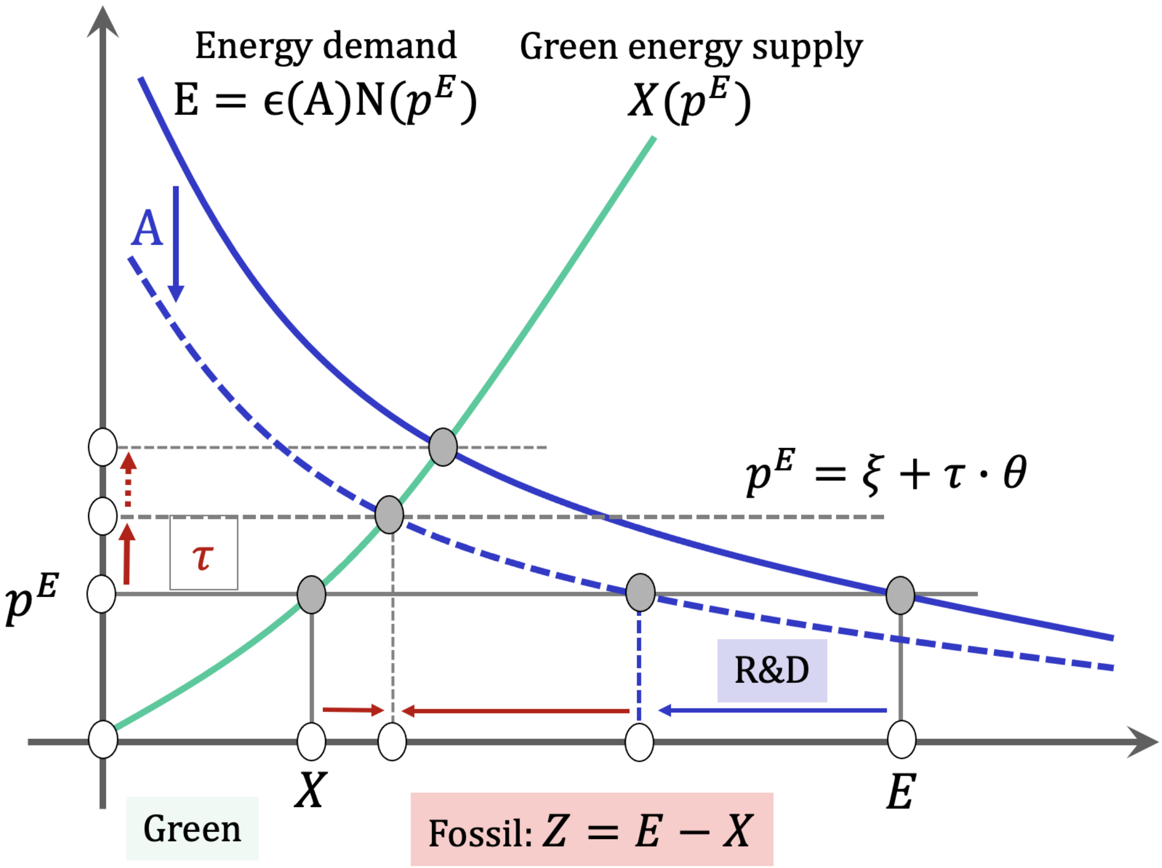
$$\chi_t^F = p_t^E F_t - (\xi + \theta\tau_t) F_t = 0 \quad \Rightarrow \quad p_t^E = \xi + \theta\tau_t. \tag{14}$$

Perfect competition equates the equilibrium market price to marginal cost of fossil fuel producers. In consequence, the fossil fuel industry serves any level of residual energy demand and makes zero profit.

Figure 3 graphically illustrates the stylized facts of the energy transition. Intuitively, energy demand declines with a higher energy price and with energy saving innovation. In a steady state, the energy coefficient is constant ($\epsilon = \epsilon^a$). Energy demand is linked to capital, $E = \epsilon N$. By (11-12), stationary investment implies $\lambda^N = 1$ and reduces expected profit of a machine to $i+1-\phi = \pi^e = \pi\phi - \epsilon p^E$ where plant-level profit gross of energy use is $\pi = \bar{\pi}\alpha y(l)$. To keep the argument simple, the second equality considers $t^y = s = 0$, suppresses management to keep the depreciation rate $1-\phi$ fixed, excludes knowledge spillovers so that $y = \bar{z}l^{1-\alpha}$, and finally keeps the energy coefficient ϵ constant. We thus consider an exogenous increase in knowledge A leading to improved energy efficiency. We show below that the labor capital ratio at the plant-level is equal to the aggregate ratio, $l = L/(\phi N)$. Given fixed labor supply L, one can invert the above condition to obtain $N = \frac{L}{\phi}\left(\frac{\bar{\pi}\alpha\bar{z}\phi}{i+1-\phi+\epsilon p^E}\right)^{1/(1-\alpha)}$. Machines require energy which adds to the user cost of capital. A higher energy price reduces investment and, in turn, energy demand.

In the absence of carbon taxes, the energy price is low, equal to the unit cost ξ in the fossil fuel sector, which reflects the abundance of fossil reserves. With low energy prices, the supply of renewable energy is limited. Low energy prices stimulate economic

Figure 3: The Energy Transition



activity, resulting in large energy demand. The major part of it is covered by fossil fuel, equal to the distance $Z = E - X$ in Figure 3. Energy saving innovation, by reducing the energy intensity of capital, shifts down the demand schedule, leading to a reduction in energy demand. At the same price $p^E = \xi$, green energy supply is unchanged. Lower demand thus crowds out fossil fuel and thereby reduces emissions. To reach 'net zero', the government can raise the carbon tax which stimulates green energy supply and further reduces demand, thereby crowding out fossil fuel, until residual demand is zero. Figure 3 also warns that the required increase in carbon taxes and energy prices would be more dramatic if innovation were absent. The resulting output losses would make the energy transition economically much more expensive.

3.4 Households

The household sector consists of a mass $\bar{L}$ of workers and a mass 1 of managers, receiving wages w_t and w_t^m. Labor supply of workers is fixed while managerial effort is endogenous. Manufacturing firms choose management intensity m_t per plant as in (6). Management pay $w_t^m = b(m_t) N_{t-1}$ must thus compensate for the effort cost of managing N_{t-1} plants. We assume current utility to be linearly separable in consumption C_t and managerial effort, giving $u(\bar{C}_t)$ with $\bar{C}_t = C_t - b(m_t) N_{t-1}$. Life-time utility in recursive form is $U_t = u(\bar{C}_t) + \beta U_{t+1}$ where $\beta < 1$ is a subjective discount factor. Non-interest income adds up to $W_t = w_t \bar{L} + \chi_t + \chi_t^X + \chi_t^F + \bar{T}_t$ in total and includes lump-sum transfers $\bar{T}_t$. The fiscal budget determines $\bar{T}_t = \tau_t \theta F_t + t_t^y (Y_t - w_t L_t) - s_t N_{t-1}$, i.e., revenue from carbon and profit taxes must pay for transfers and the investment subsidy.

Any wealth on top of firm ownership is invested in internationally traded bonds D_t which generates interest i_t on net foreign assets. The budget constraint, including management pay, is $D_t = (1 + i_{t-1}) D_{t-1} + W_t + w_t^m - C_t$. The family pools income and chooses consumption and savings to maximize life-time utility. Taking m_t as given, the solution of $U(D_{t-1}) = \max_{C_t} u(\bar{C}_t) + \beta U(D_t)$ determines consumption growth,[9]

$$u'(\bar{C}_t) = \beta (1 + i_t) \cdot u'(\bar{C}_{t+1}) . \tag{15}$$

Once current welfare is determined, consumption of goods is equal to $C_t = \bar{C}_t + b_t N_{t-1}$.

The desired effort chosen by manufacturing firms can be implemented with a linear management contract. Expand the dynamic budget constraint by $b_t N_{t-1}$ and use $\bar{C}_t$ to write $D_t = (1 + i_{t-1}) D_{t-1} + W_t + w_t^m - b_t N_{t-1} - \bar{C}_t$. Given a linear contract $w_t^m = \tilde{w}_t s_t - w_t^0$, managers maximize their surplus, $\max_{m_t} \tilde{w}_t m_t - w_t^o - b(m_t) N_{t-1}$, and choose effort such that $\tilde{w}_t = b'(m_t) N_{t-1}$. To satisfy this incentive constraint, firms must offer performance pay $\tilde{w}_t$ to implement the desired effort m_t as determined in (11.iii). Assuming that managers

[9] Define $dU_t/dD_{t-1} \equiv \lambda_t$. Necessary conditions $u'(\bar{C}_t) = \beta \lambda_{t+1}$ and $\lambda_t = (1 + i_{t-1}) \beta \lambda_{t+1}$ determine consumption growth. Specifying $u(\bar{C}_t) = \dfrac{\bar{C}_t^{1-1/\sigma^c} - 1}{1 - 1/\sigma^c}$ gives $\bar{C}_t = \bar{C}_{t+1} / ((1 + i_t) \beta)^{\sigma^c}$, where σ^c is the intertemporal elasticity of substitution.

have a zero outside option, firms cut the base salary w_t^0 until the participation constraint $w_t^m \geq b_t N_{t-1}$ is tight. The base salary is $w_t^0 = \tilde{w}_t m_t - b_t N_{t-1}$. Since the base salary is chosen to extract all surplus, management pay is $w_t^m = b_t N_{t-1}$.

3.5 Equilibrium

Manufacturers start with N_{t-1} plants. Management intensity determines survival and failure rates. Firms thus continue with $K_t = \phi_t N_{t-1}$ plants, each producing a quantity y_t of a specialized input. Given symmetry, technology $Y_t = K_t^{1/(1-\sigma)} \left[\int_0^{K_t} \left(y_t^i \right)^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}$ determines gross output of final goods equal to

$$Y_t = y_t \phi_t N_{t-1}, \quad L_t = l_t \phi_t N_{t-1}. \tag{16}$$

Total employment adds up to L_t. Output can also be represented by an aggregate production function. Multiply $y_t = z_t l_t^{1-\alpha}$ by K_t and use linear homogeneity to obtain $Y_t = z_t L_t^{1-\alpha} K_t^{\alpha}$, where $l_t = L_t / K_t$ is the labor capital ratio.

Part of gross output Y_t is absorbed by the energy sector which needs inputs $J_t + \xi F_t$ to generate energy. GDP is equal to net output $Y_t^n = Y_t - J_t - \xi F_t$. Investment demand for final goods is $I_t^D \equiv I_t + \psi_t + R_t$, and includes equipment investment $I_t + \psi_t$ and R&D spending R_t. The country's trade deficit and current account are

$$D_t = (1 + i_{t-1}) D_{t-1} - M_t, \quad M_t = C_t + I_t^D - Y_t^n. \tag{17}$$

Household invest savings in internationally traded bonds which earn interest $i_{t-1} D_{t-1}$. In a steady state, interest on net foreign assets pays for a trade deficit $M = iD > 0$.

World interest is fixed at i^*. We introduce a country premium $\zeta(d_t)$ which implies modified interest parity $i_t = i^* + \zeta_t$. When savings grow too large and the net asset ratio exceeds a benchmark, $d_t \equiv D_t / Y_t^n > \bar{d}$, the country premium falls, $\zeta'(d_t) < 0$. Domestic interest i_t thus declines and thereby slows down savings to reverse the trend. We specify

$$i_t = i^* + \zeta(d_t), \quad \zeta(d_t) = \sigma^i \cdot \left(1 - e^{d_t - \bar{d}}\right). \tag{18}$$

The premium is negatively sloped, $\zeta'(d_t) = -\sigma^i e^{\bar{d} - d_t} < 0$, and thereby stabilizes national savings. If discounting is β everywhere, stationary consumption in the world economy requires $(1 + i^*)\beta = 1$. National saving eventually leads to $i = i^*$ when $d_t \to \bar{d}$.

Equilibrium requires market clearing in final goods, labor and energy. Since green and dirty energy are perfect substitutes, there is only one market for energy,

$$Y_t^n = C_t + I_t^D - M_t, \quad \bar{L} = L_t = l_t \phi_t K_{t-1}, \quad E_t = X_t + F_t. \tag{19}$$

The energy market always clears since fossil fuel covers any residual energy demand. By Walras' Law, labor market clearing implies output market clearing.

3.6 National Climate Policy

Climate change is a global problem that requires worldwide coordination. A small country can neither affect the climate nor the outcome of international negotiations. Climate negotiations specify goals, i.e., a quantity Q_t of admissible emissions. All countries are asked to comply with the 'net zero' goal, $Q_t \to 0$, and should do so with a desired speed. We thus assume that a small country commits to an emissions reduction plan

$$Q_t = \rho^q Q_{t-1}, \quad \theta F_t = Q_{t-1}. \tag{20}$$

The plan imposes a constraint $\theta F_t \leqslant Q_{t-1}$, i.e., emissions must not exceed the quota Q_{t-1}. The national climate plan is summarized by the parameter $\rho^q < 1$. Emissions must be reduced until they are (asymptotically) zero. The parameter ρ^q determines the speed of emissions reductions and expresses 'climate ambitions'. Starting with current emissions Q_0, the quota is reduced to $Q_t = (\rho^q)^t Q_0$ after t periods.[10]

Emissions result from the use of fossil fuel. The emissions reduction plan thus requires to use all policy levers that succeed to phase out the use of fossil fuel with the least damage to the national economy.[11] The key result is that it takes more than a carbon tax to optimally phase out fossil fuel. In fact, Figure 3 suggests that using a carbon tax alone is a very costly policy to achieve the net zero goal. In the Appendix, we calculate a Pareto optimum subject to the carbon constraint (20) and find the policy levers that are required to decentralize the optimum in market equilibrium. The key instrument is the carbon tax that must be set to satisfy the constraint $Q_{t-1} = \theta F_t$ at all dates. The constraint limits the use of fossil fuel to $F_t = \epsilon^a_{t-1} N_{t-1} - X_t$. By raising the energy price to $J'(X_t) = p^E_t = \xi + \theta \tau^*_t$, the tax stimulates the supply of green energy and restricts energy demand until the residual need for fossil fuel meets the carbon constraint, as discussed in the Appendix subsequent to (A.7),[12]

$$X\left(p^E_t\right) = \epsilon^a_{t-1} N_{t-1} - Q_{t-1}/\theta \quad \Rightarrow \quad p^E_t = \xi + \theta \tau^*_t \quad \Rightarrow \quad \tau^*_t. \tag{21}$$

As the quota Q_t gets tighter, the government must raise the carbon tax ever more until emissions are net zero, $Q = \theta Z = 0$. When the use of fossil fuel is fully phased out, the supply of renewable energy must serve all demand, $X\left(p^E_t\right) = \epsilon^a_{t-1} N_{t-1}$. Figure 3 illustrates the logic. In standard integrated assessment models of a global economy, the emissions reduction plan would be part of the optimal solution, conditional on climate damages and the choice of the social discount rate. The purpose of the tax is to internalize negative externalities from the use of fossil fuel. A small open economy, however, has

[10]The time needed to cut emissions in half follows from $(\rho^q)^t = .5$. Specifying a half-life $t_{1/2}$ pins down $\rho^q = .5^{1/t_{1/2}}$. For example, cutting emissions in half within 15 years requires $\rho^q = 0.955$, meaning that the country must succeed to cut emissions by $1 - \rho^q$, i.e., by 4.5% each year.

[11]Our analysis is only concerned with the problem of how to minimize the costs of reducing emissions to zero, and doesn't account for output losses caused by global warming. One could easily consider such scenarios by taking exogenous projections of global warming and adding a damage function that translates global warming into output losses (captured by a reduction of total factor productivity, as in Nordhaus and Boyer (2000) or Golosov et al. (2014), for example).

[12]Using the supply function $X_t = \bar{x}\left(p^E_t\right)^\mu$ gives $\tau^*_t = \left[\left(\left(\epsilon^a_{t-1} N_{t-1} - Q_{t-1}/\theta\right)/\bar{x}\right)^{1/\mu} - \xi\right]/\theta$.

only a negligible impact on global warming. With an exogenously imposed emissions reduction plan, the carbon quota is like a rival public good. If a firm produces emissions and uses more of the common pool, other firms must emit less to satisfy the constraint. The purpose of the carbon tax is to internalize the negative carbon externality.

Energy saving R&D adds an additional margin of adjustment to cushion the cost of high carbon taxes. Imposing a carbon tax increases energy prices and thereby stimulates energy saving R&D which dampens the negative effect of the tax. Our analysis points to market frictions that might hamper the implementation of energy saving knowhow. To achieve net zero emissions with the lowest economic cost thus requires a complementary investment subsidy combined with a profit tax (see A.7 in the Appendix):

$$s_t^* = \omega \cdot y_t \phi_t, \quad \left(1 - t_t^{y*}\right) \bar{\pi} = 1 \quad \Leftrightarrow \quad t_t^{y*} = \left(\bar{\pi} - 1\right) / \bar{\pi}. \tag{22}$$

The only R&D externality relates to product design and investment in new plants. The task of the subsidy s_t is to compensate for knowledge spillovers as in (5). The profit tax corrects for inefficiently slow scrapping of old plants in the creative destruction cycle. Due to monopolistic rents, markets generate profits in excess of a competitive return on capital which makes firms stay too long in the market. Liquidation is too low since firms invest too much managerial effort to keep existing plants alive to appropriate those rents for a longer time. The profit tax can extract rents, leading to optimal continuation decisions. To get a sense of magnitude, consider calibrated values of the mark-up factor $\rho = 1.25$ and the cost share of capital $\alpha = 1/3$ which imply a profit markup of $\bar{\pi} = 7/5$, see (4). Extracting the monopolistic part of capital income thus requires an optimal profit tax of roughly 29%. No further correction is needed.[13]

By raising both investment and scrapping, the policy in (22) speeds up the cycle of creative destruction and accelerates the speed of diffusion of new technology. We believe this to be an important result of general significance. It should be of special relevance for the energy transition. The vintage capital model implies inertia in energy demand.

[13]Since the labor constraint implies $l_t = \bar{L} / \left(\phi_t N_{t-1}\right)$, optimal liquidation and investment also implies optimal plant-level employment.

The energy needs of a machine are fixed for its remaining life-time. Hence, the average energy efficiency of the capital stock changes only slowly, depending on the scrapping of old energy consuming machinery and its replacement by new, energy saving equipment. The speed of technological diffusion is particularly important in light of the net-zero-by-2050 goal. The faster is the diffusion of new technology, the easier it is to meet the zero emissions goal, and the lower is the cost of the energy transition. It is also worth mentioning that the fiscal cost of the optimal policy might be quite small since the profit tax pays for the cost of the investment subsidy. Although carbon tax revenue vanishes in the long-run equilibrium with zero emissions, it generates valuable tax revenue in the early part of the transition. The policy might even be self-financing.

4 Quantitative Analysis

4.1 Calibration

The quantitative model is implemented with annual frequency based on data reflecting long-run averages of the German economy. Parameter values are either calibrated, reflecting independent econometric evidence, or estimated in a model consistent way using Bayesian methods. The world interest is 2.6%, implying a discount factor of $\beta = 1/(1+i)$ in a steady state (SS). The intertemporal substitution elasticity is typically $\sigma^c = 0.5$. A markup of 25% ($\rho = \frac{\sigma}{\sigma - 1} = 1.25$) corresponds to an elasticity of variety substitution equal to $\sigma = 5$. The cost share of capital is $\alpha = 0.3$. A standard value of the depreciation (exit) rate of 10% gives a survival rate of $\phi = 0.9$. Except for the carbon tax, we set all taxes and subsidies to zero initially, $t^y = s = 0$. To achieve the net zero goal, the government must implement an emissions reduction plan. Setting $\rho^q = .9$ (the government's 'climate ambitions') implies that current emissions are reduced to less than 1% in 44 years. Table 1 reports the values and sources of calibrated parameters.

Energy prices were $p^E = 689.55$ per 1 ton of CO2 emissions in Germany in 2022 (own calculations, based on Kaltenegger et al. (2017), Andrew and Peters (2024) and

Table 1: Calibrated Parameters

Parameter	Variable	Value	Source
Capital income share	α	0.3	Gutiérrez et al. (2021)
Markup	ρ	1.25	Gutiérrez et al. (2021)
Investment spillovers	ω	0.143	Estimated
Elasticity management cost	η	0.27	Estimated
Knowledge depreciation	δ	0.908	Schankerman (1998)
R&D output elasticity	γ	0.632	Estimated
Elasticity energy efficiency	ν	1.918	Estimated
Green supply elasticity	μ	3.594	Estimated
Adjustment costs	$\bar{\psi}$	0.46	Cooper and Haltiwanger (2006)
World interest	i^*	0.026	Average 1-year US rate 2019-2024[a]
Debt to GDP ratio	$\bar{d}$	0.091	Eurostat (2024c)
Intertemp.substitution el.	σ^c	0.5	Acemoglu et al. (2012)
Emissions reduction rate	ρ^q	0.9	own

[a]From: Federal Reserve Bank of St. Louis Data (2024)

Eurostat (2024b)). Carbon costs consist of three components: (i) The European emissions trading scheme ETS covered 46% of German carbon emissions (based on German National Emissions Trading System (nEHS) (2024b)). In 2023, the average price for emissions allowances was 83.24 EUR/tCO2 based on International Carbon Action Partnership (2024) (ICAP) data. This accounts for $.46 \cdot 83.24$ of carbon costs; (ii) Germany's own national emissions allowances market covers 39% of emissions, and prices were fixed at 30 EUR/tCO2 in 2023 (National Emissions Trading System (nEHS), 2024a) which adds $.39 \cdot 30$ to carbon costs. (iii) By Eurostat (2024a), 33.4% of energy demand is for oil. Petrol and diesel excise tax revenues in 2023 were EUR 33.2 billion (Destatis, 2024), and oil-based CO2 emissions were 248.18 mt CO2 in Germany (Andrew and Peters, 2024). The petrol and diesel average excise tax is thus 133.77 EUR/tCO2 ($= \frac{33.2}{0.24818}$). The total carbon tax in Germany for 2023 is the weighted average of these components, $0.46 \cdot 83.24 + 0.39 \cdot 30 + 0.334 \cdot 133.77 = 105.71$ EUR/tCO2. The carbon tax (including ETS prices) is 15% of the energy price (per ton of carbon, $\tau\theta/p^E = 105.1/689.5$).

German CO2 emissions were $Q = 665.6$ megatons in 2022, based on Andrew and Peters (2024), and GDP was 4121 bn Euro in 2023, $Y^n = 4121$ (Eurostat, 2024b). It is convenient to normalize GDP to $Y^n = 100$. To preserve GDP ratios and the carbon tax rate, we rescale emissions and energy prices, giving $Q \approx 16.15$ and $p^E \approx 1.673$. Knowing the energy price and the tax liability (in rescaled units) implies a zero profit production cost $\xi = p^E - \tau\theta$ in the fossil fuel sector. GDP and gross output are related by $Y^n = Y - J - \xi F$. Calibration computes gross output Y consistent with a cost share $s^E = .11$ of energy as reported by Kaltenegger et al. (2017) for Germany in 2011, assuming that it remained constant since then. Energy cost is thus $p^E E = s^E Y$. Eurostat (2024a) data for Germany indicate that 22% of energy was green (renewables and nuclear) in 2022, $X = s^X E$ with $s^X = .22$. The remainder is fossil fuel, $F = \left(1 - s^X\right) E$. Given carbon emissions of Q, the emissions coefficient must be $\theta = Q/F$. We estimate the price elasticity of green energy supply in (13) equal to $\mu = 3.594$, taking the estimate of 3.6 by Lamp et al. (2024) as a prior, see the discussion of Table 2 below. Multiplying the f.o.c. by X, we find $J = p^E X / (1 + 1/\mu)$ and set $\bar{x}$ to support these quantities.

In a SS, average and marginal energy intensities are identical, equal to $\varepsilon = \varepsilon^a = E/Y$. Given cost shares, the survival rate ϕ, and a normalized wage $w = 1$, we calibrate plant-level output y and the number of varieties N to support gross output $Y = y\phi N$. We treat investment and R&D in new product lines for specialized inputs as one process. Our estimate for the spill-over rate in (5) is $\omega = 0.143$, based on a prior of $\omega = 0.15$. The estimate of ω rationalizes the optimal subsidy rate for R&D and investment in manufacturing, and accords well with observed subsidy rates. The results of Cooper and Haltiwanger (2006) imply $\bar{\psi} = 0.46$ which determines investment smoothing. A key channel is the endogenous effect of management intensity on survival and destruction rates. We specify a linear success probability $\phi(m) = \phi_0 + m$ with a basic success rate of ϕ_0, and a convex increasing cost function $b(m) = \bar{b}\frac{m^{1+\eta}}{1+\eta}$. Referring to the optimality condition (11.iii), we calibrate the convexity parameter such that a 5% higher continuation value per plant boosts the survival rate by 2 percentage points ($d\phi = .02$, e.g., from .9 to .92). This serves as a prior of $\eta = .25$ for the estimation procedure, and results in an estimated value of $\eta = .27$, see Table 2.

Firms invest in energy saving R&D to expand know-how by $\varphi(R) = \bar{\varphi}\left(R + \bar{R}\right)^{\gamma}$.

Accumulated knowledge A, in turn, determines the design of new machines with energy intensity $\epsilon(A) = \bar{e}\frac{A^{1-\nu}}{\nu-1}$. Taking the mean value of the estimates of Schankerman (1998), knowledge becomes obsolete (depreciates) at rate $\delta = 0.908$. The findings of Dechezleprêtre et al. (2016) suggest that a 1% higher R&D incentive (e.g., a reduction in user cost, or higher patent prices, as measured by a higher shadow price λ^A) boosts innovation output A by 2.6% and implies an elasticity of knowledge creation of $\gamma = 0.615$. The estimation below results in a slightly higher value of 0.635 which is reasonable compared to the results of Dechezleprêtre et al. (2016). Parameter ν controls for the impact of innovation on energy intensity. We rely on Chen et al. (2024) whereby a 1% higher innovation output reduces energy intensity E/Y of production by 0.09%. This implies that average energy intensity ϵ^a changes by 0.09%. As investments (in the initial steady state) are 10% of capital stock, marginal energy intensity ϵ needs to change by $0.09/0.1 = 0.9\%$ to affect the average energy intensity by 0.09%. Such a change in marginal intensity implies $\nu = 1.9$ which we use as a prior for estimation. The final estimation yields $\nu = 1.918$, relatively close to the value estimated by Chen et al. (2024).

A small open economy faces a given world interest rate i^*. Germany's net foreign assets were 9.1% of GDP in 2023 (Eurostat, 2024c). With infinitely lived families, the national interest rate must adjust to stabilize savings. Following Uribe and Schmitt-Grohé (2017), we thus assume a modified interest parity condition $i = i^* + \zeta(d)$ where the country premium as defined in (18) depends negatively on the net asset ratio. We set the elasticity of the interest premium to be small, $\sigma^i = .005$, which implies that domestic interest i falls by 5 basis points (relative to i^*, from 3% to 2.95%) if foreign assets increase by 10 pp of GDP (from $d = .1$ to 0.2).

4.2 Estimation

We estimate key parameters that are not commonly known: knowledge spillovers ω in manufacturing output and investment; the elasticity of R&D output γ in energy saving innovation; the elasticity of energy intensity ν; the price elasticity μ of green energy supply; and the convexity parameter η of the managerial cost function. We also introduce five shocks to make the model track real world data, and estimate the persistence and standard deviations of these shocks. For that purpose, we augment the model by

$$
\begin{aligned}
\text{Productivity:} \quad & \varepsilon_{z,t} = \rho_z \varepsilon_{z,t-1} + \sigma_{z,t}, \qquad & p_t &= e^{\varepsilon_{z,t}} \bar{z} N_{t-1}^{\omega}, \\
\text{Depreciation:} \quad & \varepsilon_{\phi,t} = \rho_\phi \varepsilon_{\phi,t-1} + \sigma_{\phi,t}, \qquad & \phi_t &= (\phi_0 + m_t) e^{\varepsilon_{\phi,t}}, \\
\text{Energy efficiency:} \quad & \varepsilon_{\epsilon,t} = \rho_\epsilon \varepsilon_{\epsilon,t-1} + \sigma_{\epsilon,t}, \qquad & \varepsilon_{\epsilon,t} &= \frac{\bar{e} A_{t-1}^{1-\nu}}{\nu - 1} e^{\varepsilon_{\epsilon,t}}, \\
\text{Capital utilization:} \quad & \varepsilon_{N,t} = \rho_N \varepsilon_{N,t-1} + \sigma_{N,t}, \qquad & N_t &= (I_t + \phi N_{t-1}) e^{\varepsilon_{N,t}}, \\
\text{Green energy:} \quad & \varepsilon_{p^E,t} = \rho_{p^E} \varepsilon_{p^E,t-1} + \sigma_{p^E,t}, \qquad & X_t &= \bar{X} p_t^E e^{\varepsilon_{p^E,t}}.
\end{aligned} \tag{23}
$$

The prior values for Bayesian estimation (see Table 2) are assumed to be normally distributed. Regarding shock processes, all priors of the persistence ρ_j are based on Beta probability density functions with a mean value of 0.5 and a standard deviation of 0.2.

To estimate the DSGE model, we use German annual data from 2000 to 2019 on real GDP Y_t^n (Eurostat, 2024b), energy use E_t (U.S. Energy Information Administration, 2023), real gross fixed capital formation I_t (World Bank, 2024), and total factory productivity z_t (Feenstra et al., 2015). All variables are converted into annual growth rates by log-differencing the data series. We apply a Hodrick-Prescott (HP) filter to de-trend the growth rate data series and choose the smoothing parameter of the HP filter to be 100 because data series employed are in annual frequency. Measurement equations are:

$$
\begin{aligned}
dY_t^n &= \log(Y_t^n) - \log(Y_{t-1}^n), \\
dE_t &= \log(E_t) - \log(E_{t-1}), \\
dTFP_t &= \log(z_t) - \log(z_{t-1}), \\
dI_t &= \log(I_t) - \log(I_{t-1}).
\end{aligned} \tag{24}
$$

The estimated parameters and their 90% highest posterior density intervals are in Table 2. Overall, parameter estimates are in line with priors. The parameter ν governs how R&D knowledge translates into energy efficiency. The empirical literature on that margin is scarce. We thus estimate the parameter and obtain a value $\nu = 1.918$ which marginally exceeds the value of 1.9 based on Chen et al. (2024).

Table 2: Parameter Estimates

Parameter	Variable	Prior mean	Posterior mean	90% HPD
Investment spillovers	ω	0.15	0.143	(0.11, 0.17)
Elasticity energy efficiency	ν	1.9	1.918	(1.87, 1.95)
R&D output elasticity	γ	0.615	0.632	(0.6, 0.66)
Green supply elasticity	μ	3.60	3.594	(3.44, 3.77)
Elasticity management cost	η	0.25	0.27	(0.25, 0.29)

Our estimate $\gamma = 0.632$ implies an R&D elasticity of 2.7%, compared to the estimate of 2.6% in Dechezleprêtre et al. (2016) who focus on relatively small firms. The survey of Becker (2015) suggests that these values are at the top end of existing estimates which mostly relate to industrial R&D. Our model, in contrast, specifically focuses on energy saving innovation. Our estimate of the price elasticity of green energy supply is similar to (Lamp et al., 2024). For ω and η, we use our own priors which are relatively close to the final estimated values.

4.3 Simulation Results

To achieve a zero emissions equilibrium, a country must phase out the use of fossil fuel. Using a carbon tax only to manage the energy transition is bound to be a costly policy that reduces economic activity due to high energy prices. Energy saving innovation and an optimal policy for a faster diffusion of new technology can significantly reduce transition costs and improve the climate economy trade-off. To highlight the importance of these arguments, we explore two scenarios. In a first scenario, the government raises the carbon tax without any complementary policies. A second scenario implements the optimal policy package, including a profit tax and an investment cum R&D subsidy as discussed in (22), on top of the carbon tax. Table 3 reports long-run effects when the transition to zero emissions is complete, with column 'Tax' referring to the first and 'OP' to the second scenario. The upper part of the Table reports absolute values and the lower part percent changes relative to the initial equilibrium.

Table 3: Long-Run Effects

Sym	Variable	ISS	Tax	OP
$\tau\theta/p^E$	Carbon Tax	0.15	0.41	0.41
t_y	Profit Tax	0	0	0.32
s	Inv.R&D Subsidy	0	0	0.1
$\bar{T}/Y^n$	Fiscal Budget	0.01	0	0
$100\varepsilon^a$	Energy Intensity	4.26	3.97	3.81
$1-\phi$	Scrapping Rate	0.1	0.09	0.12
p^E, %	Energy Price		43.21	43.95
π^e, %	Exp. Profit/Plant		-5.12	14.86
π, %	Gross Profit/Plant		8.33	6.89
X, %	Green Energy		263.58	270.35
E, %	Energy Demand		-20.01	-18.52
N, %	Capital		-14.16	-8.9
Y, %	Gross Output		-6.34	-4.65
Y^n, %	Net Output, GDP		-7.75	-6.14
I^D, %	Total Investment		-13.93	11.55
C, %	Consumption		-6.21	-10.55
$\bar{C}$, %	Current Welfare		-6.31	-5.27

4.3.1 Carbon Tax Only

When the energy transition is complete, energy supply stems from renewable sources only. All fossil fuel is replaced. Consequently, carbon tax revenue vanishes and transfers to households are zero. Crowding out fossil fuel requires the carbon tax to rise from 15 to 41% of energy prices.[14] Energy prices are thus 43% higher than in the initial equilibrium. Firms react on several margins. On impact, and in the absence of tax, higher energy prices reduce expected profit $\pi^e = \pi\phi - \epsilon^a p^E - b$ per plant. In a steady state, by (11), expected profit must match interest plus depreciation $\pi^e = i + 1 - \phi$. Since real interest is constant across steady states, a higher survival rate, or a lower scrapping rate $1-\phi$, limits the decline in expected profit and thus requires a compensating increase in gross profit per plant, π. Management intensity follows from (11.iii) which reduces

[14] We estimate the current carbon tax in Germany at 15.3% of the energy price which corresponds to 105.7 EUR per ton of CO2 emitted. On the way to net zero, the carbon tax must rise to 41% of the energy price. This implies a tax liability of 403.68 Euro per ton of CO2 emitted (408.8 Euro with optimal policy). These figures are in line with the literature, although slightly on the higher end (Nordhaus, 1991; Golosov et al., 2014; Nordhaus, 2017; Barrage, 2018).

to $b'(m) = \phi'(m)(1+\pi)$ in a steady state where the investment cum R&D subsidy and the profit tax are both zero. The rise in gross profit induces an increase in management intensity to benefit from a continued profit stream per plant that is kept alive. Thanks to better management, the survival rate increases and thereby reduces the scrapping rate $1-\phi$. The need to earn a higher gross profit $\pi = \bar{\pi}\alpha y$ induces a decline in investment. Plant-level output and employment, y and l must both increase. With fixed labor supply, the resource constraint $L = l\phi N$ much restricts steady state capital N.[15]

Quantitatively, a 43% increase in energy prices induces large reactions. Expected profit per plant declines by more than 5.1%, consistent with a one percentage point lower scrapping rate. To limit the decline in expected profit to this level, gross plant-level profit π must rise by more than 8%, requiring an equally large increase in plant-level output, and rising plant-level employment. The labor constraint restricts capital accumulation which shrinks the number of plants by 14.2% in the long-run. Gross output $Y = y\phi N$ declines by 6.3% which is the net result of a structural shift to fewer but larger plants. Given the increase in management intensity, the larger management costs result in an even larger decline of -7.8% in net output (GDP).

The rather strong increase in energy prices is needed to restrict demand and stimulate the supply of green energy. Currently, the major part of energy stems from the use of fossil fuel. The price increase must thus stimulate a large expansion of green energy supply which is up by 264% relative to current levels. At the same time, the high prices greatly increase the shadow price λ^E of energy use and the value λ^A of energy saving knowledge in (12). Accordingly, manufacturers invest more in energy saving R&D, see (11.ii), which boosts knowhow A and leads to a more energy efficient design in new machines, $\epsilon'(A) < 0$. In a steady state, average and marginal energy intensity are the same, shrinking from 4.26% of capital initially to 3.97% which corresponds to a reduction of roughly 7%. Total energy demand $E = \epsilon N$ thus declines by 20% in Table 3, not only due to less investment but also thanks to increased energy efficiency.

[15] Alternatively, one might define a user cost of capital including the energy cost per machine. Higher energy prices, by raising the operating costs of machines, would then increase the user cost of capital and thereby depress investment.

Since adjustment costs vanish in a steady state, total investment $I^D = I + R$ reflects investment I in new equipment (cum R&D in new product lines) as well as energy saving R&D. The reduction in equipment investment $I = (1-\phi)N$ is proportional to the number of plants (each embodying one unit of capital) which declines by 14.2% for that reason. However, the lower scrapping rate of existing plants reduces the need for replacement investment which makes the percent reduction in total investment smaller. On the other hand, increased spending R on energy saving R&D augments total investment. Since it represents only a small share of total investment, the impact is small. The net effect of all three channels is a strong decline in total investment of 13.9%. The drop in investment exceeds the GDP reduction of 7.8%. Accordingly, private consumption shrinks by 6.2% which falls short of the net output loss. The higher survival rate of plants prevents a larger reduction of GDP but also requires a larger compensation of management effort. The loss of current welfare $\bar{C} = C - b(m)N$ thus exceeds the decline in consumption.

A scenario using a carbon tax only to achieve the net zero goal is bound to be economically costly. The loss in welfare, of course, must be viewed in light of the country's desirable contribution to stop global warming. In a global model with climate damages, the marginal welfare gains from a better climate with lower damages would balance the economic costs of an optimal carbon tax. Our analysis of a small open economy excludes such benefits, and is exclusively concerned with the problem of how to achieve the energy transition with the least economic cost.

4.3.2 Optimal Policy

The optimal policy maximizes national welfare subject to the carbon constraint imposed by the commitment to an internationally coordinated emissions reduction plan. The government could manage the energy transition in a better way by complementing the carbon tax with a profit tax combined with an investment subsidy as in (22). Manufacturers overinvest in management and continue old plants for too long to exploit monopolistic rents on differentiated inputs. The profit tax corrects for this distortion and induces a larger exit of incumbent plants. On the other end of the creative destruction cycle, the design and introduction of new product lines benefit from knowledge spillovers from accumulated research experience which leads to too little business creation. To correct for this positive externality, the government offers an investment cum R&D subsidy which stimulates investment in new and better machines. Existing machines created in a period of low energy prices prior to the energy transition also consume a lot of energy. By accelerating both exit and entry of plants, the policy speeds up the creative destruction cycle, contributes to a faster diffusion of new energy saving technology, and thereby is able to reduce the economic cost of the energy transition.

The direct effect of the profit tax is to stimulate exit. The scrapping rate increases to 12%, up by 3 percentage points relative to the 'Tax' scenario. To cover interest and depreciation costs, manufacturers must thus earn substantially higher expected profit per plant, $\pi^e = i+1-\phi$, which is 14.9% higher relative to the status quo. This increase reflects the optimal ex post investment subsidy $s = .1$, equal to 10% of the acquisition cost of equipment (one unit of capital) per plant. It also results from a substantial reduction in management costs $b(m)$ as manufactures cut back on management intensity and thereby choose a higher scrapping rate when gross profit gets taxed at a higher rate. Not only is the profit tax optimally increased from zero to 32%, the gross cash-flow per plant rises somewhat less than in the 'Tax' scenario. The subsidy and the savings in management costs dominate the reduction in net of tax cash-flow per plant $(1 - t^y)\pi$. The lower cash-flow reflects smaller employment and output per plant and accelerates exit of old plants. Combined with a lower survival rate, the labor constraint $L = l\phi N$ requires a much larger investment and number of plants N, consistent with a larger expected profit π^e per plant. The policy shifts production to more but smaller establishments and, on net, cushions the decline in GDP from -7.8 to -6.1%.

The carbon tax hardly changes and the increase in the energy price relative to the 'Tax' only scenario is very small. Accordingly, the expansion of green energy supply is only marginally larger. However, the policy induces a significant increase in energy saving R&D. Given a much larger number of plants (N increases by 5.3 pp relative to the 'Tax' scenario), the shadow price λ^E of average energy intensity in (12.iii) must rise relative to the 'Tax' scenario, thereby increasing the value of energy saving knowhow. Firms thus spend significantly more on energy saving R&D which leads to improved energy efficiency of the capital stock, with the energy intensity ϵ^a of machines further declining from 4 to 3.8%. The expansionary nature of optimal policy implies larger energy demand relative to the 'Tax' scenario (i.e., $E = \epsilon^a N$ declines by -18.5% only instead of -20%) which is small compared to the large increase in the number of establishments. The improved energy efficiency of capital prevents a larger increase in energy demand relative to 'Tax'.

The largest adjustments in response to the optimal policy complements are in the use of GDP which results from the acceleration of creative destruction. The larger scrapping rate requires much higher replacement investment to keep the capital stock constant. In addition, manufacturers also need to invest more in energy saving R&D. For both reasons, aggregate investment I^D now increases by 11.6% relative to the status quo, instead of a 13.9% decline in the 'Tax' only scenario. The much higher investment needs crowd out private consumption of goods even though GDP is 1.6 percentage points higher. However, lower consumption comes together with a much reduced effort cost of management. On net, the current welfare measure $\bar{C}$ improves. After all, optimal policy is maximizing welfare. Current welfare thus declines by only 5.3 instead of 6.3%.[16] We conclude that optimal policy, by combining a carbon tax with a profit tax and an ex post investment subsidy, can significantly reduce the economic cost of the energy transition.

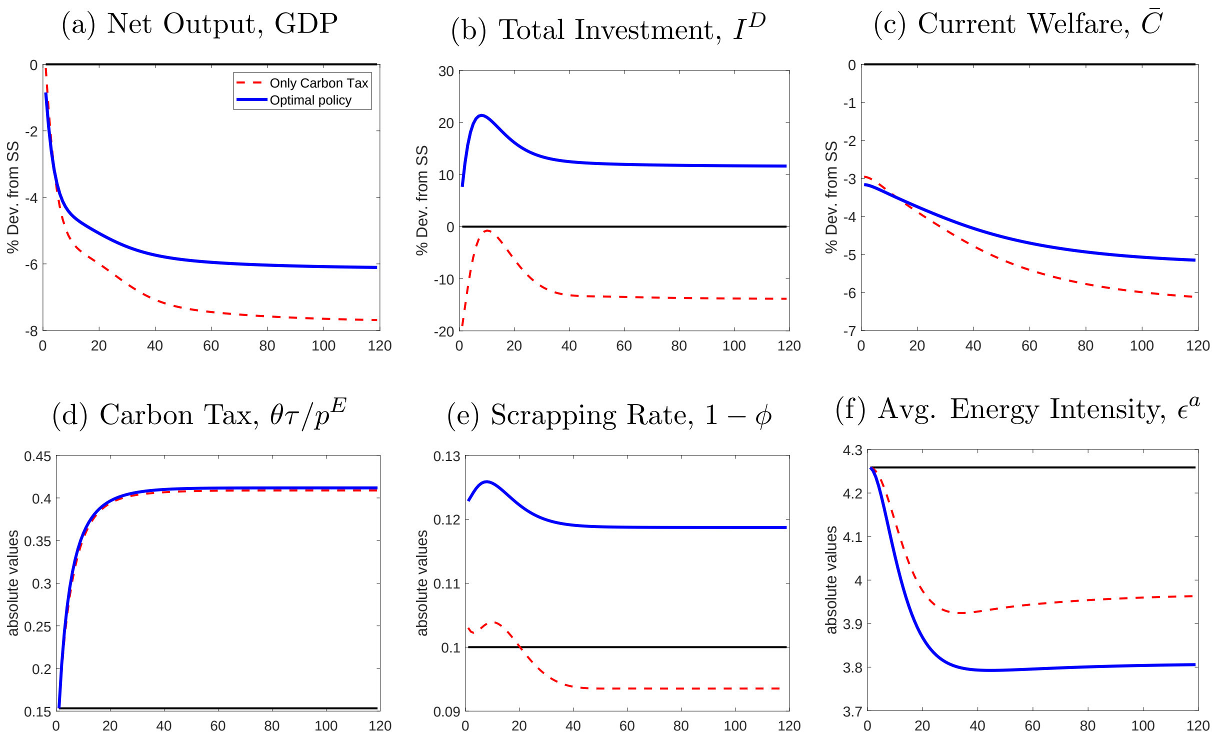
4.3.3 Dynamic Adjustment

Figure 4 illustrates transition paths over 120 years, starting from the status quo to the long-run zero emissions equilibrium. The economic impact of the energy transition evolves slowly, reflecting the speed of capital accumulation and the slow diffusion of energy saving technology in the vintage capital model. The red broken lines refer to the carbon 'Tax' only scenario while the blue solid lines indicate adjustment under the fully optimal policy which complements the carbon tax with a corrective profit tax and an ex post investment

[16] A true welfare measure would have to consider the present value of welfare changes along the entire transition path.

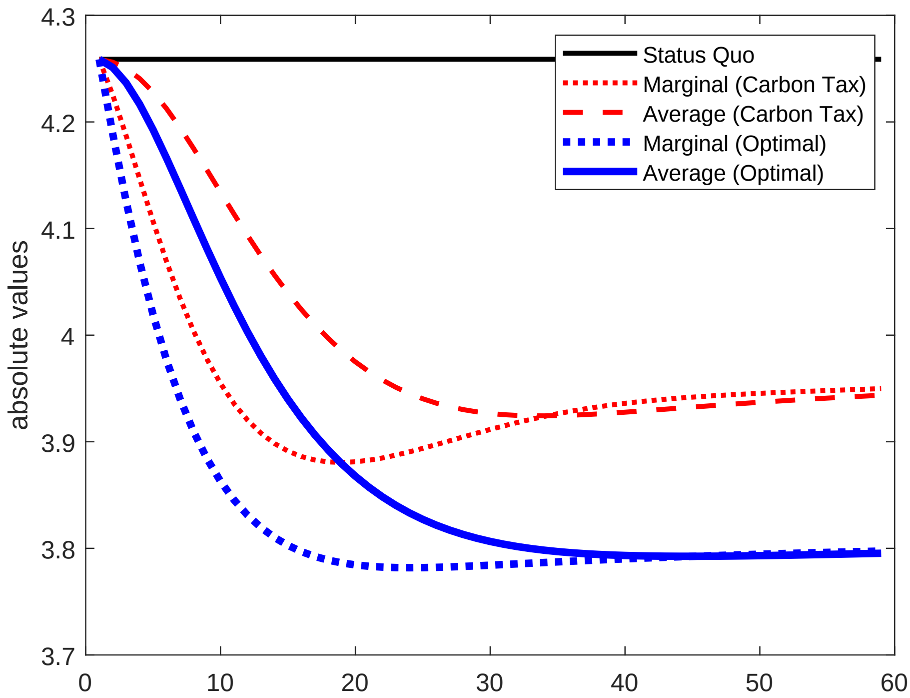
subsidy. The time series ultimately approach the long-run values reported in Table 3. The Figure thus illustrates the same qualitative results except that short-run effects tend to be smaller in magnitude. By assumption, management intensity is adjusted without friction, implying that the scrapping rate immediately jumps up and, in fact, overshoots long-run adjustment. Since more scrapping requires higher replacement investment for old plants, aggregate investment overshoots as well.

Figure 4: Carbon Tax and the Optimal Energy Transition



The flow of current welfare $\bar{C}$ indicates that the 'carbon tax only' scenario is slightly less costly in a first adjustment phase. The reason is the slow implementation of the emissions reduction plan as in (20). The government wishes to reduce emissions and phase out fossil fuel at a slow rate. Accordingly, the carbon tax liability increases only with much delay and is small in the short-run. In consequence, there is little pressure to increase the scrapping rate which rises only to minor extent in the short-run and is reduced later on in the 'Tax' only scenario. Managerial effort cost thus moderately shrinks in the short-run and then rises over time as the scrapping rate is reduced and the survival rate accordingly increases. In consequence, current welfare losses mainly reflect the short-run decline in consumption and get magnified over time with increasing effort costs of management. With a fully optimal policy, the carbon tax is gradually increased as before but the profit tax combined with ex post investment subsidy are more or less instantaneously implemented. The scrapping rate is much higher at all dates and overshoots in the short-run. All together, the adjustments in consumption and managerial effort cost imply that, in a first phase, welfare losses are larger than in the carbon tax only scenario, and become smaller thereafter and in the entire future. Since optimal policy maximizes welfare subject to the carbon constraint, the net effect must be a welfare gain when the energy transition starts, i.e., the present value of the periodic welfare flow must be positive.

Figure 5: Average (ϵ^a) and Marginal (ϵ) Energy Intensity



Finally, Figure 5 illustrates the slow diffusion of energy saving technological progress in a vintage capital model. The blue lines refer to the fully optimal scenario, the red lines refer to the carbon tax only case. In the long-run, average and marginal energy intensities are the same, and the lines converge to the values reported in Table 3. The marginal energy intensity of new machines declines very fast. High energy prices stimulate energy saving R&D which leads to a more energy efficient design of new machines. The average energy intensity of the capital stock declines only to the extent that new and more efficient machines replace old equipment, and is much slower to adjust.

5 Conclusion

To stop global warming, all countries worldwide must commit to phase out fossil fuel to rapidly reduce carbon emissions. A small open economy cannot significantly affect global warming and the resulting climate damages. Yet they must commit to an internationally coordinated emissions reduction plan to achieve a state of net zero emissions in a few decades. The problem of a small country is to minimize the economic burden of managing an energy transition towards net zero.

Energy saving innovation can play an important role to mitigate the costs of the green transition. In a vintage capital model, the price elasticity of energy demand is small in the short-run and much larger in the long-run. In the same vein, the benefits of innovation also depend on the rate of diffusion of technological progress. In this context, our analysis emphasizes that exclusively relying on a carbon tax only is bound to be a very costly policy to manage the energy transition. A fully optimal policy complements the carbon tax with a profit tax and an investment subsidy. These two complementary policies induce a larger scrapping rate of old equipment and, at the same time, a larger investment rate to faster install new and more energy saving equipment. These policies thereby speed up the cycle of creative destruction, leading to a faster turnover of capital and a more rapid diffusion of energy saving technology. Using a DSGE model with estimated and calibrated parameters, we find that an energy transition using a carbon tax only imposes substantial economic costs, with a GDP loss of roughly 7.8% in the long-run compared to the status quo trend. Moving to the optimal policy could limit the GDP loss to roughly 6.1%.

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Appendix I: Pareto Optimum

International negotiations determine a carbon quota for each country. A small economy can neither affect the emissions quota nor world market prices. This Appendix calculates a Pareto optimum subject to the carbon constraint (20) and then determines the policy levers that can decentralize the optimal allocation in market equilibrium.

A Pareto optimum maximizes welfare $U_t = u\left(\bar{C}_t\right) + \beta U_{t+1}$ subject to the current account (17), $D_t = (1 + i_{t-1}) D_{t-1} + Y_t^n - I_t^D - C_t$, and subject to labor, energy and carbon constraints. Using $\bar{C}_t = C_t - b_t N_{t-1}$, substituting for Y_t^n and I_t^D, and finally noting final goods output $Y_t = y_t \phi_t N_{t-1}$ to rewrite the current account gives the planner's dynamic resource constraint $D_t = (1 + i_{t-1}) D_{t-1} + W_t - \bar{C}_t$ with income net of effort cost equal to $W_t = (y_t \phi_t - b_t) N_{t-1} - J_t - \xi F_t - I_t - \psi_t - R_t$.

Static Optimization: Total energy use is $E_t = \epsilon_{t-1}^a N_{t-1}$. We first maximize net income W_t subject labor, energy and carbon constraints. Lagrange multipliers of the constraints are w_t^*, p_t^{E*} and τ_t^*. We solve

$$\begin{aligned} W_t &= \max_{l_t, X_t, F_t} \left[y\left(l_t, z_t\right) \phi\left(m_t\right) - b\left(m_t\right) \right] N_{t-1} - J\left(X_t\right) - \xi F_t - \left[I_t + \psi\left(I_t, N_{t-1}\right) + R_t \right] \\ &: \quad -w_t^* \cdot \left[l_t \phi_t N_{t-1} - \bar{L} \right] - p_t^{E*} \cdot \left[\epsilon_{t-1}^a N_{t-1} - X_t - F_t \right] + \tau_t^* \cdot \left[Q_{t-1} - \theta F_t \right]. \qquad \text{(A.1)} \end{aligned}$$

Optimality conditions yield maximum net income $W_t = W\left(m_t, R_t, I_t, N_{t-1}, \epsilon_{t-1}^a, Q_{t-1}\right)$:

$$l_t : w_t^* = y_l\left(l_t, z_t\right), \quad X_t : p_t^{E*} = J'\left(X_t\right), \quad F_t : \xi + \theta \tau_t^* = p_t^{E*}. \qquad \text{(A.2)}$$

Dynamic Optimization: The planner maximizes welfare $U_t = u\left(\bar{C}_t\right) + \beta U_{t+1}$ subject to the resource constraint $D_t = (1 + i_{t-1}) D_{t-1} + W_t - \bar{C}_t$. The maximized level of income net of effort cost, $W_t = W\left(m_t, R_t, I_t, N_{t-1}, \epsilon_{t-1}^a, Q_{t-1}\right)$, follows from (A.1). The planner recognizes knowledge spillovers $z_t = \bar{z}\left(N_{t-1}\right)^\omega$. The laws of motion are $N_t = I_t + \phi\left(m_t\right) N_{t-1}$ for optimal accumulation capital (number of plants), $A_t = \varphi\left(R_t\right) + \delta A_{t-1}$ for energy saving knowledge, $\epsilon_t^a = \left[\epsilon\left(A_{t-1}\right) I_t + \phi\left(m_t\right) \epsilon_{t-1}^a N_{t-1}\right] / N_t$ for average energy intensity of capital as in (7), and $Q_t = \rho^q Q_{t-1}$ for the emissions reduction plan. The value function $U_t = U\left(D_{t-1}, N_{t-1}, A_{t-1}, \epsilon^a_{t-1}, Q_{t-1}\right)$ depends on predetermined state variables. A Pareto optimal intertemporal allocation must solve the Bellmann problem $U_t = \max_{\bar{C}_t, I_t, R_t, m_t} u\left(\bar{C}_t\right) + \beta U\left(D_{t-1}, N_{t-1}, A_{t-1}, \epsilon^a_{t-1}, Q_{t-1}\right)$. Use shadow prices $\tilde{\lambda}^N_t \equiv \frac{dU_t}{dN_{t-1}}$, $\tilde{\lambda}^A_t \equiv \frac{dU_t}{dA_{t-1}}$, $\tilde{\lambda}^D_t \equiv \frac{dU_t}{dD_{t-1}}$ and $\tilde{\lambda}^Q_t \equiv \frac{dU_t}{dQ_{t-1}}$. Since ϵ^a_{t-1} reduces value, we define $\tilde{\lambda}^E_t \equiv -\frac{dU_t}{d\epsilon^a_{t-1}}$. Optimality conditions are

$$\begin{aligned}
\bar{C}_t &: \; u'\left(\bar{C}_t\right) = \beta \tilde{\lambda}^D_{t+1}, \quad R_t : -W_{R_t} \cdot \beta \tilde{\lambda}^D_{t+1} = \varphi'\left(R_t\right) \beta \tilde{\lambda}^A_{t+1}, \\
I_t &: \; -W_{I_t} \cdot \beta \tilde{\lambda}^D_{t+1} = \beta \tilde{\lambda}^N_{t+1} - \frac{d\epsilon^a_t}{dI_t} \beta \tilde{\lambda}^E_{t+1}, \\
m_t &: \; 0 = W_{m_t} \cdot \beta \tilde{\lambda}^D_{t+1} + \phi'\left(m_t\right) \left[N_{t-1} \beta \tilde{\lambda}^N_{t+1} - \frac{d\epsilon^a_t}{d\phi_t} \beta \tilde{\lambda}^E_{t+1} \right].
\end{aligned} \tag{A.3}$$

Envelope conditions are

$$\begin{aligned}
D_{t-1} &: \; \tilde{\lambda}^D_t = \left(1 + i_{t-1}\right) \beta \tilde{\lambda}^D_{t+1}, \quad A_{t-1} : \tilde{\lambda}^A_t = \delta \beta \tilde{\lambda}^A_{t+1} - \epsilon'\left(A_{t-1}\right) \frac{d\epsilon^a_t}{d\epsilon_t} \beta \tilde{\lambda}^E_{t+1}, \\
N_{t-1} &: \; \tilde{\lambda}^N_t = W_{N_{t-1}} \cdot \beta \tilde{\lambda}^D_{t+1} + \phi_t \beta \tilde{\lambda}^N_{t+1} - \frac{d\epsilon^a_t}{dN_{t-1}} \beta \tilde{\lambda}^E_{t+1}, \\
\epsilon^a_{t-1} &: \; \tilde{\lambda}^E_t = -W_{\epsilon^a_{t-1}} \cdot \beta \tilde{\lambda}^D_{t+1} + \frac{d\epsilon^a_t}{d\epsilon^a_{t-1}} \beta \tilde{\lambda}^E_{t+1}, \quad Q_{t-1} : \tilde{\lambda}^Q_t = W_{Q_{t-1}} \cdot \beta \tilde{\lambda}^D_{t+1} + \rho^q \beta \tilde{\lambda}^Q_{t+1}.
\end{aligned} \tag{A.4}$$

Combining (A.3.i) and (A.4.i) gives optimal consumption growth,

$$u'\left(\bar{C}_t\right) = \left(1 + i_t\right) \beta \cdot u'\left(\bar{C}_{t+1}\right). \tag{A.5}$$

Since the planner is subject to the same interest rates as households, consumption growth in market equilibrium is optimal, see (15).

To compare with relevant market conditions, we divide (A.3-A.4) by $u'\left(\bar{C}_t\right) = \beta \tilde{\lambda}^D_{t+1}$, define $\lambda^{N*}_t \equiv \frac{\tilde{\lambda}^N_t}{u'\left(\bar{C}_t\right)}$ and similarly λ^{E*}_t, λ^{A*}_t, and λ^{Q*}_t, and use $\frac{1}{1+i_t} = \frac{\beta u'\left(\bar{C}_{t+1}\right)}{u'\left(\bar{C}_t\right)}$ as in (A.5). By the envelope theorem, the derivatives of the maximized value (A.1) are $W_{R_t} = -1$, $W_{I_t} = -\left(1 + \psi_{I,t}\right)$, and $W_{m_t} = \left[\left(y_t - w_t l_t\right) \phi'\left(m_t\right) - b'\left(m_t\right)\right] N_{t-1}$ where

$y_t - w_t l_t = \pi_t^*$. Using this, and computing changes of ϵ_t^a as mentioned in (7), results in

$$\begin{aligned} R_t &: \ 1 = \varphi'(R_t) \frac{\lambda_{t+1}^{A^*}}{1+i_t}, \quad I_t : 1 + \psi_{I,t} = \frac{\lambda_{t+1}^{N^*}}{1+i_t} + \frac{\epsilon_t^a - \epsilon_t}{N_t} \frac{\lambda_{t+1}^{E^*}}{1+i_t}, \\ m_t &: \ b'(m_t) = \phi'(m_t) \left[\pi_t^* + \frac{\lambda_{t+1}^{N^*}}{1+i_t} - \frac{\epsilon_{t-1}^a - \epsilon_t^a}{N_t} \frac{\lambda_{t+1}^{E^*}}{1+i_t} \right]. \end{aligned} \tag{A.6}$$

Applying again the envelope theorem to the problem (A.1) yields $W_{\epsilon_{t-1}^a} = -p_t^{E*} N_{t-1}$ and $W_{Q_{t-1}} = \tau_t^*$. The planner observes knowledge spillovers $z_t = z(N_{t-1})$. We thus have $W_{N_{t-1}} = (y_t + y_{z,t} z_t' N_{t-1} - w_t^* l_t) \phi_t - b_t - \psi_{N_{t-1}} - \epsilon_{t-1}^a p_t^{E*}$, where $y_t - w_t^* l_t = \pi_t^*$. Envelope conditions are thus transformed to

$$\begin{aligned} A_{t-1} &: \ \lambda_t^{A^*} = \delta \frac{\lambda_{t+1}^{A^*}}{1+i_t} - \epsilon'(A_{t-1}) \frac{I_t}{N_t} \frac{\lambda_{t+1}^{E^*}}{1+i_t}, \\ N_{t-1} &: \ \lambda_t^{N^*} = W_{N_{t-1}} + \phi_t \left[\frac{\lambda_{t+1}^{N^*}}{1+i_t} - \frac{\epsilon_{t-1}^a - \epsilon_t^a}{N_t} \frac{\lambda_{t+1}^{E^*}}{1+i_t} \right], \\ &: \ W_{N_{t-1}} = \pi_t^* \phi_t - b_t - \psi_{N_{t-1}} - \epsilon_{t-1}^a p_t^{E*} + y_{z,t} z_t' N_{t-1} \phi_t, \\ \epsilon_{t-1}^a &: \ \lambda_t^{E*} = p_t^{E*} N_{t-1} + \phi_t \frac{N_{t-1}}{N_t} \frac{\lambda_{t+1}^{E*}}{1+i_t}, \quad Q_{t-1} : \lambda_t^{Q*} = \tau_t^* + \rho^q \frac{\lambda_{t+1}^{Q*}}{1+i_t}. \end{aligned} \tag{A.7}$$

Can public policy steer market equilibrium towards a Pareto optimum?

Decentralization: The market allocation is optimal if private choices are structurally identical to (A.2) and (A.6-A.7). Comparing (13-14) with (A.2.ii-iii) shows that a carbon tax τ_t^* is required to induce an optimal energy allocation, $\xi + \theta \tau_t^* = p_t^{E*} = J'(X_t)$. The energy and carbon constraints in (A.1), $Q_{t-1} = \theta F_t = \theta \left(\epsilon_{t-1}^a N_{t-1} - X_t \right)$, pin down X_t and, in turn, p_t^{E*} and τ_t^*. When the two allocations are identical, the market price correctly indicates scarcety of energy, $p_t^E = p_t^{E*}$. By (A.7.iii) and (12.iii), shadow prices are identical as well, $\lambda_t^E = \lambda_t^{E*}$.

Due to monopolistic rents, profits exceed optimal levels. By (4), private plant-level profit is $\pi_t = y_t - w_t l_t = \bar{\pi} \alpha y_t$ while social profit is $\pi_t^* = \alpha y_t$ (multiply the f.o.c. $w_t^* = y_l$ by l_t and use $l_t y_{l,t} = (1-\alpha) y_t$ in $\pi_t^* = y_t - w_t^* l_t$) which implies

$$\pi_t = \bar{\pi} \cdot \pi_t^*, \quad \bar{\pi} = 1 + \frac{\rho - 1}{\rho} \frac{1 - \alpha}{\alpha} \geq 1. \tag{A.8}$$

Substitute π_t^e in (8) into the market valuation of capital (12.i) and compare with (A.7.ii):

$$\begin{aligned}
\lambda_t^{N*} &= \pi_t^* \phi_t - b_t - \psi_{N_{t-1}} - \epsilon_{t-1}^a p_t^{E*} + y_{z,t} z_t' N_{t-1} \phi_t + \phi_t \left[\frac{\lambda_{t+1}^{N*}}{1+i_t} - \frac{\epsilon_{t-1}^a - \epsilon_t^a}{N_t} \frac{\lambda_{t+1}^{E*}}{1+i_t} \right], \\
\lambda_t^N &= (1 - t_t^y)\, \pi_t \phi_t - b_t - \psi_{N_{t-1}} - \epsilon_{t-1}^a p_t^E + s_t + \phi_t \left[\frac{\lambda_{t+1}^N}{1+i_t} - \frac{\epsilon_{t-1}^a - \epsilon_t^a}{K_t} \frac{\lambda_{t+1}^E}{1+i_t} \right]. \quad \text{(A.9)}
\end{aligned}$$

Note $p_t^E = p_t^{E*}$ and $\lambda_{t+1}^E = \lambda_{t+1}^{E*}$ by the argument above. The solutions of (A.9) are identical if $(1 - t_t^y)\, \pi_t = \pi_t^*$ and $s_t = y_{z,t} z_t' N_{t-1} \phi_t$. Use $z_t' = \omega z_t / N_{t-1}$ by (5), $z_t y_{z,t} = y_t$ by (3), and note $\pi_t = \bar{\pi} \pi_t^*$ in (A.8). The optimal policy is thus

$$s_t^* = \omega \cdot y_t \phi_t, \quad (1 - t_t^{y*})\, \bar{\pi} = 1 \quad \Leftrightarrow \quad t_t^{y*} = (\bar{\pi} - 1) / \bar{\pi}. \qquad \text{(A.10)}$$

The optimal profit tax induces optimal managerial effort and scrapping. To see this, note $\pi_t^* = y_t - w_t^* l_t$ and compare (11.iii) with (A.6.iii),

$$\begin{aligned}
m_t^* &: \; b'(m_t) = \phi'(m_t) \left[\pi_t^* + \frac{\lambda_{t+1}^{N*}}{1+i_t} - \frac{\epsilon_{t-1}^a - \epsilon_t^a}{K_t} \frac{\lambda_{t+1}^{E*}}{1+i_t} \right], \qquad \text{(A.11)} \\
m_t &: \; b'(m_t) = \phi'(m_t) \left[(1 - t_t^y)\, \pi_t + \frac{\lambda_{t+1}^N}{1+i_t} - \frac{\epsilon_{t-1}^a - \epsilon_t^a}{N_t} \frac{\lambda_{t+1}^E}{1+i_t} \right]
\end{aligned}$$

The tax reduces profits to the optimal level, $(1 - t_t^y)\, \pi_t = \pi_t^*$. Since $p_t^E = p_t^{E*}$ and $\lambda_{t+1}^E = \lambda_{t+1}^{E*}$ with an optimal carbon tax, and since (A.9) implies $\lambda_{t+1}^N = \lambda_{t+1}^{N*}$, the policy also induces optimal management, $m_t = m_t^*$, and, in turn, optimal scrapping, $\phi_t = \phi_t^*$.

No further correction is needed. Given $\lambda_{t+1}^N = \lambda_{t+1}^{N*}$ and $\lambda_{t+1}^E = \lambda_{t+1}^{E*}$, investment policies in (A.6.ii) and in (11.i) are identical, implying $I_t = I_t^*$. In the same vein, (A.6.i) and (11.ii) are identical, $R_t = R_t^*$, since the shadow prices λ_{t+1}^A and λ_{t+1}^{A*} in (A.7.i) and (12.ii) are identical as well.

Appendix II: DSGE Model Estimation Procedure

The estimation procedure determines parameter values to match model variables with observed data. The log-likelihood function indicates goodness-of-fit. Using the Kalman filter, we approximate the log-likelihood function of observing the actual German data in stochastic model simulations conditional on given parameter values. With the Kalman filter, one computes the likelihood that the observed data emerge from the state-space representation of the model.

The state-space representation consists of the transition equations, which describe the evolution of the state variables, and the measurement equations, which relate the state variables to the observed data. The transition equations are $\mathbf{s}_t = \mathbf{A}\mathbf{s}_{t-1} + \varepsilon_t$ where $\mathbf{s}_t$ is the vector of state variables, $\mathbf{A}$ is the transition matrix, and ε_t is the vector of shocks. Knowing the current state variables, we can back out our variables of interest from measurement equations (24). Using the Kalman filter, we recursively update the state variables and compute the log-likelihood of the observed data. The Metropolis-Hastings algorithm is then used to sample from the posterior distribution of the parameters, the prior distributions and the likelihood function. In this way, we produce parameter posterior distributions which are summarized in Table 2.