Centre for International Macroeconomic Studies
School of Economics, University of Surrey

Summer School:

Advanced Macro-Modeling

Central Bank Communication, Imperfect Credibility, and Optimal Monetary Policy Applications

Ricardo Nunes
Don Park
Luca Rondina

CONTENTS

1 INTRODUCTION

This course is aimed at researchers and policy makers with some knowledge of Dynamic Stochastic General Equilibrium (DSGE) macroeconomic models. The material covers applications and the underlying theory of optimal policy. We will discuss issues of central bank communication, targeting rules, and monetary policy design. The course will also address time-inconsistency and analyze a concept of imperfect credibility that spans the polar cases of the time-inconsistent and time-consistent solutions. We will both address the theoretical and practical aspects of imperfect commitment. The course will focus mainly on monetary policy but the tools are broader and can be applied to other frameworks, e.g. fiscal or macro-prudential policy.

This is a hands-on course based on Matlab and Dynare. The aim is that participants can incorporate such analysis in their research, economic analysis, and policy work. In this respect, we will be providing a toolkit in the form of Matlab codes. This toolkit is well documented, easy to use, and can be applied to large scale models as well.

1.1 INSTRUCTORS

Ricardo Nunes is a Professor in the School of Economics at the University of Surrey. He graduated from Universitat Pompeu Fabra (Barcelona, Spain) obtaining a MSc in Economics in 2003 and a PhD in Economics in 2007. After graduating he spent 10 years in the Federal Reserve System under various roles. In 2007 he joined the Board of Governors of the Federal Reserve System, where he worked as an economist and senior economist. In 2014 he moved to the Federal Reserve Bank of Boston working as a senior economist and policy advisor. He was also a visiting researcher at the Bank of Portugal and the IMF and has given talks at various central banks. In February 2018 he was appointed to the Council of Economic Advisers to the Chancellor of the Exchequer. His main research is on monetary and fiscal policy. He has published extensively in these areas including the Quarterly Journal of Economics, Journal of Monetary Economics, Journal of Economic Theory, Journal of European Economic Association, Journal of International Economics, among others.

Don Park is a PhD candidate at the University of Surrey. He has a MSc in economics from the University of Surrey and a BA from Seoul National University. He worked at the Bank of Korea as a Junior Economist from 2008-2014 and as an Economist from 2014-2016. His research focuses on macroeconomics, monetary policy, inflation expectations and consumption behaviour.

Luca Rondina is a Lecturer in Economics at the University of Sussex. He obtained a PhD degree at the University of Surrey in 2019. He graduated from the Politecnico di Milano in Industrial Engineering obtaining a BSc in 2012 and a MSc in 2015. His research focuses on macroeconomics, taxation, and labor supply.

1.2 TIME-TABLE

The time-table is as follows:
9.30 - 11.00 am: Session 1
11.00 - 11.30 am: Coffee and Tea
11.30 am -1.00 pm: Session 2
1.00 - 2.15 pm: Lunch
2.15-3.45 pm: Session 3
3.45 - 4.15 pm: Coffee and Tea
4.15 - 5.45pm : Session 4

1.3 COURSE CONTENTS

The contents of lectures given by the instructors over the one day are as follows:

- Simplest New Keynesian model:
  - Time-inconsistent solution
  - Time-consistent solution
  - This will be derived and explained in a manner consistent with recursive contracts theory and will set the stage for the imperfect commitment settings.
- Central bank communication:

  - State-contingent nature of commitment
  - Targeting rules
- Monetary policy design
  - Benefits of price level targeting
  - Alternative policies
- Imperfect credibility
- Some theory behind solutions
- Toolkit of imperfect credibility
- Application to large scale models
  - Smets and Wouters AER 2007 model
    * What are the gains of achieving more credibility?
    * How does the possibility of future re-optimizations affect current outcomes and promises?
    * How does imperfect credibility affect the shock propagation, volatilities, and cross-correlations between relevant variables?
    * Does the policy response to some shocks require more commitment? At what stages?

2 BIBLIOGRAPHY FOR THE COURSE

2.1 MAIN BIBLIOGRAPHY

- Debortoli, D., Maih, J., and Nunes, R., 2014. Loose Commitment in Medium-Scale Macroeconomic Models: Theory and Applications, Macroeconomic Dynamics. vol. 18, pp. 175-198.

- Ben S. Bernanke 2017. Monetary Policy in a New Era, Brookings Institution, October 2, 2017.

- Bodenstein, M. Hebden, J, Nunes, R. 2012 Imperfect Credibility and the Zero Lower Bound on the Nominal Interest Rate, Journal of Monetary Economics, vol. 59, pp. 135-149.

- Clarida, R., Gali, J., Gertler, M., 1999. The science of monetary policy: A new keynesian perspective. Journal of Economic Literature 37 (4), 1661-1707.

- Marcet, A., Marimon, R., 2009. Recursive contracts. Universitat Pompeu Fabra. Working Paper.

- Campbell, Jeffrey R., Jonas D. M. Fisher, Alejandro Justiniano, and Leonardo Melosi, 2016, Forward guidance and macroeconomic outcomes since the financial crisis, in NBER Macroeconomics Annual 2016.

- Currie, D. and Levine, P. 1993. Rules, Reputation and Macroeconomic Policy Coordination. Cambridge University Press.

- Evans, G. and Honkapohja, Seppo. 2003. Expectations and the Stability Problem for Optimal Monetary Policies, Review of Economic Studies, Oxford University Press, vol. 70, pages 807-824.

- Evans, G. and Honkapohja, Seppo. 2006. Monetary Policy, Expectations and Commitment, Scandinavian Journal of Economics, Wiley Blackwell, vol. 108, pages 15-38.

- Walsh, C. "Monetary Theory and Policy" Third Edition, MIT Press, Chapter 7.

2.2 ADDITIONAL READINGS BY TOPIC

2.2.1 Imperfect credibility and Recursive Contracts

- Ball, L., 1995. Disinflation with imperfect credibility. Journal of Monetary Economics 35 (1), 5–23.
- Debortoli, D., Nunes, R., 2010. Fiscal policy under loose commitment. Journal of Economic Theory 145 (3), 1005-1032.
- Debortoli, D., Maih, J., Nunes, R., 2014. Loose Commitment in Medium-Scale Macroeconomic Models: Theory and Applications. Macroeconomic Dynamics. vol. 18, Issue 01, pp. 175-198.
- Roberds, W., 1987. Models of policy under stochastic replanning. International Economic Review 28 (3), 731-755.
- Schaumburg, E., Tambalotti, A., 2007. An investigation of the gains from commitment in monetary policy. Journal of Monetary Economics 54 (2), 302-324.

2.2.2 Communication and Commitment

- Campbell, Jeffrey R., Jonas D. M. Fisher, Alejandro Justiniano, and Leonardo Melosi, 2017, "Forward guidance and macroeconomic outcomes since the financial crisis," in NBER Macroeconomics Annual 2016, Martin Eichenbaum and Jonathan A. Parker (eds.), NBER Macroeconomics Annual, Vol. 31, Chicago: University of Chicago Press, pp. 283-357.
- Discretion solution: Dennis, R., February 2007. Optimal policy in rational expectations models: New solution algorithms. Macroeconomic Dynamics 11 (01), 31-55.
- Soderlind, P., 1999. Solution and estimation of re macromodels with optimal policy. European Economic Review 93, 813-823.

- Klein, P., Krusell, P., Rios-Rull, J.-V., 2005. Time consistent public policy. Review of Economic Studies 75, 789-808.

2.2.3 Optimal Policy Implementation

- Evans, G. and Honkapohja, Seppo. 2003. Expectations and the Stability Problem for Optimal Monetary Policies, Review of Economic Studies, Oxford University Press, vol. 70(4), pages 807-824.
- Evans, G. and Honkapohja, Seppo. 2006. Monetary Policy, Expectations and Commitment, Scandinavian Journal of Economics, Wiley Blackwell, vol. 108(1), pages 15-38, March.

2.2.4 Zero lower bound

- Adam, K., Billi, R., 2006. Optimal monetary policy under commitment with a zero bound on nominal interest rates. Journal of Money, Credit and Banking 38 (7), 1877-1905.
- Adam, K., Billi, R., 2007. Discretionary monetary policy and the zero lower bound on nominal interest rates. Journal of Monetary Economics 54 (3), 728-752.
- Billi, R., 2009. Optimal inflation for the U.S. economy. Federal Reserve Bank of Kansas City Working Papers RWP 07-03.
- Bodenstein, M., Erceg, C., Guerrieri, L., 2009. The effects of foreign shocks when interest rates are at zero. International Finance Discussion Papers 983.
- Bodenstein, M. Hebden, J, Nunes, R. 2012 Imperfect Credibility and the Zero Lower Bound on the Nominal Interest Rate, Journal of Monetary Economics, vol. 59, Issue 2, pp. 135-149.
- Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, Pedro Teles. 2013 Unconventional Fiscal Policy at the Zero Bound. American Economic Review.
- Eggertsson, G. B., 2006. The deflation bias and committing to being irresponsible. Journal of Money, Credit and Banking 38 (2), 283-321.

- Eggertsson, G. B., Woodford, M., 2003. The zero bound on interest rates and optimal monetary policy. Brookings Papers on Economic Activity 34 (1), 139-233.
- Erceg, C., Linde, J., 2010. Is there a fiscal free lunch in a liquidity trap?, mimeo, Federal Reserve Board.
- Levin, A., Lopez-Salido, D., Nelson, E., Yun, T., 2010. Limitations on the effectiveness of forward guidance at the zero lower bound. International Journal of Central Banking 6 (1), 143–189.
- Nakov, A., 2008. Optimal and simple monetary policy rules with zero floor on the nominal interest rate. International Journal of Central Banking 4 (2), 73-127.
- Reifschneider, D., Williams, J., 2000. Three lessons for monetary policy in a low inflation era. Journal of Money, Credit, and Banking 32, 936-966.

2.2.5 Delegation schemes and optimal monetary policy design

- Ben S. Bernanke 2017. Monetary Policy in a New Era, Brookings Institution, October 2, 2017.
- Jensen, H., 2002. Targeting nominal income growth or inflation? American Economic Review 92 (4), 928-956.
- McCallum, B. T., 1995. Two fallacies concerning central-bank independence. American Economic Review 85 (2), 207-11.
- Rogoff, K., 1985. The optimal degree if commitment to an intermediate monetary target. Quarterly Journal of Economics 100 (4), 1169-89.
- Strum, B. (2010) "Monetary Policy in a Forward-Looking Input-Output Economy," Journal of Money, Credit and Banking, vol. 41 (June 2009), pp. 619-50.
- Svensson, L., 1997. Optimal inflation contracts, conservative central banks, and linear inflation contracts. American Economic Review 87 (1), 98-114.
- Vestin, D., 2006. Price-level versus inflation targeting. Journal of Monetary Economics 53 (7), 1361-1376.

- Walsh, C., 1995. Optimal contracts for central bankers. American Economic Review 85 (1), 150-167.
- Walsh, C. E., 2003b. Speed limit policies: The output gap and optimal monetary policy. American Economic Review 93 (1), 265-278.
- Woodford, M., 2003b. Optimal interest-rate smoothing. Review of Economic Studies 70 (4), 861-886.

2.2.6 Multiple solutions and discretionary equilibria

- Alstadheim, R. and Røisland, Ø. (2017), When Preferences for a Stable Interest Rate Become Self-Defeating. Journal of Money, Credit and Banking, 49.
- Blake, A. P., Kirsanova, T., 2010. Discretionary policy and multiple equilibria in lq re models. manuscript, University of Exeter.
- Dennis, R., Kirsanova, T., 2010. Expectations traps and coordination failures: selecting among multiple discretionary equilibria. Working Paper Series 2010-02, Federal Reserve Bank of San Francisco.

2.2.7 Timeless perspective

- Benigno, P., Woodford M. 2006 Linear-Quadratic Approximation of Optimal Policy Problems
- Debortoli, D., Nunes, R., 2006. On linear-quadratic approximations. Universitat Pompeu Fabra. Manuscript. http://mpra.ub.uni-muenchen.de/544/
- Christian Jensen and Bennett T. McCallum (2002) "The Non-Optimality of Proposed Monetary Policy Rules Under Timeless Perspective Commitment" Economics Letters 77, October 2002; 147-300.
- Dennis, Richard (2010) "When is discretion superior to timeless perspectivepolicymaking?", In Journal of Monetary Economics, Volume 57, Issue 3, 2010, Pages 266-277.
- Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

2.2.8 Targeting rules

- Giannoni, M., Woodford, M., 2010. Optimal target criteria for stabilization policy. NBER Working Paper 15757.
- Svensson, L. 1999 "Inflation Targeting as a Monetary Policy Rule," Journal of Monetary Economics 43 607-654

2.2.9 Academic articles on Central Banks' policy:

- Goodfriend, M., 2010. Central banking in the credit turmoil: An assessment of Federal Reserve practice. Journal of Monetary Economics forthcoming.
- King, R. G., 2010. Discussion of ''limitations on the effectiveness of forward guidance at the zero lower bound". International Journal of Central Banking 6 (1), 191–203.
- Svensson, L., 2009. Transparency under flexible inflation targeting: Experiences and challenges. Sveriges Riksbank Economic Review 1, 5–44.
- Svensson, L., 2010. Policy expectations and policy evaluations: The role of transparency and communication. Sveriges Riksbank Economic Review forthcoming.
- Sveriges Riksbank, 2009. Material for assessing monetary policy. www.riksbank.com, 1–58.
- Walsh, C., 2009. Using monetary policy to stabilize economic activity. UC Santa Cruz, manuscript.

2.2.10 Sustainable plans and reputation mechanisms

- Barro, R. J., 1986. Reputation in a model of monetary policy with incomplete information. Journal of Monetary Economics 17 (1), 3–20.
- Backus, D., Drifill, J., 1985. Inflation and reputation. American Economic Review 75 (3), 530-38.
- Chari, V. V., Kehoe, P. J., 1990. Sustainable plans. Journal of Political Economy 98 (4), 783-802.

- Currie, D., Levine, P., 1993. Rules, Reputation and Macroeconomic Policy Coordination. Cambridge Books. Cambridge University Press.
- Sleet, C., 2001. On credible monetary policy and private government information. Journal of Economic Theory 99 (1-2), 338-376.
- Reis, C., 2008. Taxation without commitment. Manuscript.

2.2.11 Applications of Time-Inconsistency to Fiscal Policy

- Aiyagari, R., Marcet, A., Sargent, T. J., Seppala, J., 2002. Optimal taxation without state-contingent debt. Journal of Political Economy 110, 1220-1254.
- Davide Debortoli and Ricardo Nunes "Lack of Commitment and the Level of Debt" International Finance Discussion Papers 938. Washington: Board of Governors of the Federal Reserve System, 2008.
- Diaz-Gimenez, J., Giovannetti, G., Marimon, R., Teles, P., 2008. Nominal debt as a burden to monetary policy. Review of Economic Dynamics 11(3), 493-514.
- Ortigueira, S., 2006. Markov-perfect taxation. Review of Economic Dynamics 9, 153-178.

2.2.12 Time-Inconsistency Classical Papers/Books:

- Barro, R. J., Gordon, D. B., 1983. A positive theory of monetary policy in a natural rate model. Journal of Political Economy 91 (4), 589-610.
- Currie, D. and Levine, P. 1993. Rules, Reputation and Macroeconomic Policy Coordination. Cambridge University Press.
- Kydland, F. E., Prescott, E. C., 1977. Rules rather than discretion: The inconsistency of optimal plans. Journal of Political Economy 85 (3), 473-91.
- Marcet, A., Marimon, R., 2009. Recursive contracts. Universitat Pompeu Fabra. Working Paper.

3 | TOOLKIT DOCUMENTATION

Toolkit Documentation of "Loose Commitment in Medium-Scale Macroeconomic Models: Theory and an Application"

Davide Debortoli	Junior Maih	Ricardo Nunes
UC San Diego	Norges Bank	Federal Reserve Board

This appendix documents the toolkit to solve loose commitment settings easily in medium- and large-scale linear-quadratic models. If you use or modify these codes, please cite the paper "Loose Commitment in Medium-Scale Macroeconomic Models: Theory and an Application".

The first section describes the installation and a simple example. The second section describes the example used in the paper. The paper replication codes allow the user to explore the possibilities and options in the toolkit quite extensively. The third section discusses the specific files in the toolkit.

The toolkit codes can be downloaded at dss.ucsd.edu/~ddebortoli/ or ricardonunes.net. These codes are written in Matlab and have been tested in version 7.7. The toolkit integrates with Dynare and has been tested with version 4.1.1. Do not add the directory to the Matlab path.

The files needed for the toolkit are contained in the main folder, while the two subfolders contains the files for the two specific examples described below.

1 Getting started

The first step consists in writing your `.dyn` file, where the model of interest is specified. At the beginning of your file, specify the location of the toolkit files, adding the line:

```
addpath('[destination folder]','-begin');
```

1.1 Example: a standard New-Keynesian model

The folder NK_example contains the files related to a simple New-Keynesian model, whose only structural equation is a standard New-Keynesian Phillips curve with an AR(1) cost-push shock driving the dynamics. The main file NK.dyn can be executed with the command dynare NK.dyn.

The model is declared as follows:

```
varexo E_Y;
var PAI OUT;
parameters ProbabilityOfCommitment;
model(linear);
   PAI = 0.99*PAI(+1)+0.1*OUT+E_Y;
end;
shocks;
   var E_Y   ; stderr .01;
end;
```

and the policymaker's objective function is declared with the commands:

```
planner_objective -.5*(1*PAI^2+0.048*OUT^2);
options_.planner_discount = 0.99;
```

Models with more lags and leads can be specified in their original formulation, since the code automatically transforms them into the compact formulation used in the paper.

The following commands initiate the loose commitment toolkit:

```
Compare=0;
if Compare;
   ramsey_policy(nograph,nomoments); //,nograph
else
   options_.loosecommit= 1;
   ProbabilityOfCommitment=.5;
   stoch_simul(nograph,nomoments,noprint);
end
```

Setting options_.loosecommitment = 1 tells the program to use the toolkit.[1]

[1] The line referring to the ramsey_policy is necessary to execute several intermediate steps even if it is not read.

The command `stoch_simul` solves the model. The solution of the model, as well other information, is stored in the structured variable `oo_`. In particular, the law of motion is summarized by the matrices `oo_.dr.ghx` and `oo_.dr.ghu`.

Once the solution has been obtained, the user can produce simulations and statistics according to her needs. For convenience, the toolkit already provides additional files to generate impulse responses, second moments, and welfare calculations. In particular:

1. Impulse response functions are executed with the commands:

   ```
   Periods2=20;   Sim_nbr=1;   InitShocks=[];   InitVals=[];
   CommitmentHistory=ones(Periods2,1);
   IRF=LooseCommitmentIrf(Periods2,Sim_nbr,InitShocks,InitVals,...
   CommitmentHistory);
   figure('name','Never reoptimization');
   plot(IRF')
   legend(M_.endo_names)
   ```

   The sample codes contain several additional examples for IRFs – where the history of commitment shocks, initial values, and scope of the IRFs are changed.

2. Moment calculations are executed with the commands:

   ```
   Periods2=500; Sim_nbr=1500; Burn=100; InitVals = [];
   CommitmentHistory=[];
   MOM=LooseCommitmentMoments(Periods2,Sim_nbr,Burn,InitVals,...
   CommitmentHistory);
   disp 'Simulated Moments'
   disp(M_.endo_names)
   disp(MOM)
   ```

3. Welfare calculations are executed with the commands:

   ```
   [UncondWelf10,CondWelf10]=LooseCommitmentWelfare;
   v=0; Y_Mu_0=rand(M_.endo_nbr,1);
   [UncondWelf11,CondWelf11]=LooseCommitmentWelfare(Y_Mu_0,v);
   ```

The first line considers the initial conditions and steady-state to be at zero. The second command computes welfare for different initial conditions.

2 The Smets and Wouters (2007) model

The folder `SW_example` contains the files used to generate the main results in the paper. The main file is `SW_main.dyn`, which starts by calling the following files:

- `SW_model.dyn`, defining the model equations and calibration;
- `SW_objective_planner_benchmark.dyn` and `SW_objective_planner_alternative.dyn`, setting the two specifications of the central bank loss function analyzed in the paper.

These two files are the only files that need to be adapted when solving a different model.

The user is then required to specify some options. In particular, the user can choose to solve the model for multiple degrees of commitment (including full-commitment and discretion). Accordingly, the core of the program iterates on the possible degrees of commitment, as follows

```
if Compare;
    ramsey_policy(nograph,nomoments) pinf y yf r; //,nograph
    disp('Problem solved using Ramsey Policy')
else
    for j = [iter:-1:1];
        ProbabilityOfCommitment=probCOM_grid(j);
        stoch_simul(nograph,nomoments,noprint);
        [...]
    end
end
```

For all the degrees of commitment – contained in the vector `probCOM_grid` – the model is solved using the file `stoch_simul.m` and, if desired, welfare, impulse response functions, and second moments are computed by calling the functions `LooseCommitmentWelfare.m`, `LooseCommitmentIrf.m`, and `LooseCommitmentMoments.m`, respectively. Finally, the output is reorganized and displayed on the screen using the file `SW_showresults.m`, which makes use of the toolkit files `plot_IRFs.m` and `show_tables.m`.

The policy frontiers and the Monte-Carlo simulations reported in the paper are produced using two similar files – `SW_frontier.m` and `SW_regression.m`, respectively.

3 File documentation of the main toolkit files

- `stoch_simul.m`: When the option `options_.loosecommit=1` in the `.dyn` file, the toolkit will be used. `stoch_simul.m` is an intermediate file that integrates the toolkit with Dynare. This file temporarily substitutes the original stoch_simul.m dynare file, and later versions will incorporate this toolkit directly.

- `LooseCommitment.m`: This file transforms the equations of the model according to the formulation in the paper, and then solves the problem (using SolveLooseCommitment.m). Options to be set:

  - `qz_criteriumLC`: determines the cutoff point to characterize an eigenvalue to be explosive (default is 1.000001);
  - `MaxIterLC`: determines the maximum number of iterations for convergence (default is 3000);
  - `critLC`: determines the the convergence criteria (default is 1e-7);
  - `noprint`: determines whether the code prints output in the command window (default is 0 for printing).

  Variables of interest: The variable `Hold` is conveniently used to store the latest solution. This is useful when solving the model for different degrees of commitment, so that an homothopy method can be exploited. `M_.endo_names` and `M_.exo_names` are character vectors with the names of the endogenous and exogenous variables, respectively.

- `SolveLooseCommitment.m`: This file is the main engine for the solution procedure. It executes the iteration loop described in the paper, and exits successfully if

$$max(H - Hold) < critLC,$$

and produces an error if the maximum number of iterations is reached, the model is unstable, or if an unspecified error occurs.

- `GetDynareLooseCommitmentResults.m`: This file conveniently reorganizes the output of Dynare (stored in `oo_`). After the solution is computed, this file recovers all the necessary information and passes it to other subcodes to compute moments, IRFs, etc. This file shows where each variable is stored in memory.

- `LooseCommitmentIrf.m`: This file computes the impulse response functions. If the inputs to the file are not passed, the file resets those but the order of inputs skipped needs to be in the correct order (see code for the specific details). Inputs to the file are:

  - `Periods`: number of periods in the simulation (default is 40).
  - `Sim_nbr`: number of simulations (default is 1000).
  - `InitShocks`: vector of initial conditions for the shocks. If this input is empty, then it is drawn stochastically. The initial condition for the specific IRF shock is always reset to one positive standard deviation.
  - `InitVals`: vector of initial conditions for the variables. If this input is empty, then it is set to a vector of zeros (assumed to be the steady-state).
  - `CommitmentHistory`: vector of re-optimization shocks at each period (0 for reoptimization, 1 otherwise). The code uses the commitment history for each simulation. If this input is empty, the commitment history is re-sampled stochastically (default is re-sampling stochastically).

  The code takes the difference between the series where the initial IRF shock is standardized to one positive standard deviation, and the series where it is set to zero. The seed of the random number generator is not set inside this function and needs to be set in the `.dyn` file (see example files). The file produces the IRF for all shocks and all series. To identify specific series by name we provide the file `find_var.m`. To plot the series, we provide the file `plot_IRFs.m`.[2]

[2] If the user wants to plot a large number of series, she will have to increase the number of line styles in the file `plot_IRFs.m`.

- `LooseCommitmentMoments.m`: This file computes the variance of the series and its structure is similar to the impulse response functions file.[3] A dissimilarity is that while the IRF file takes the difference between two series (with and without the initial shock), the moments file does not. Inputs to the file are:
  - `Periods, InitVals, Sim_nbr`, and `CommitmentHistory`: options equal to file `LooseCommitmentIrf.m`.
  - `Burn`: number of periods to discard in the simulation (default is 100). Total number of periods in the simulation is given by input Periods (not Periods minus Burn).
  - `shocks_sel`: turns off some shocks in case the corresponding element is set to zero (default is vector of ones). This option is useful to compute conditional moments and variance decompositions.

  Option InitShocks is not used, and shocks are always stochastic.

- `LooseCommitmentWelfare.m`: This file computes conditional and unconditional (on the initial shocks) welfare. The corrections discussed in the paper when $\lambda_{t-1} \neq 0$ are incorporated. Inputs to the file are:
  - `Y_Mu_0`: vector of initial conditions for the variables. If this input is empty, then it is set to a vector of zeros.
  - `v`: vector of initial conditions for the shocks. If this input is empty, then it is set to a vector of zeros.

[3]This file can be easily adapted to produce averages or other moments.

4 PAPER DEBORTOLI, NUNES, MAIH (2014)

Loose Commitment in Medium-Scale Macroeconomic Models: Theory and Applications*

Davide Debortoli
University of California San Diego

Junior Maih
International Monetary Fund

Ricardo Nunes
Board of Governors of the Federal Reserve System

Abstract

This paper proposes a method and a toolkit for solving optimal policy with imperfect commitment. As opposed to the existing literature, our method can be employed in medium- and large-scale models typically used in monetary policy. We apply our method to the Smets and Wouters (2007) model, where we show that imperfect commitment has relevant implications for interest rate setting, the sources of business cycle fluctuations, and welfare.

JEL *classification*: C32, E58, E61.
Keywords: Commitment, Discretion, Monetary Policy

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1 Introduction

In the modern macroeconomic literature, economic outcomes result from the interactions between policymakers and rational firms and households. A common feature of these models is that economic decisions (e.g. consumption, hours worked, prices) depend on expectations about future policies (e.g. taxes, interest rates, tariffs). As shown by Kydland and Prescott (1977) optimal policy plans in this class of models are subject to time-inconsistency.

The modern literature has taken different approaches to address this problem. One possibility is to assume that policymakers can fully commit – a single optimization is undertaken and the chosen policies are then implemented in all subsequent periods. This approach is known as full-commitment or simply commitment. An alternative, often referred to as discretion or no-commitment, assumes that policymakers cannot commit and that policy plans always need to be time-consistent. Although many types of time-consistent equilibria can be studied, one of the most common approaches is to solve for Markov-perfect equilibria, where policy functions only depend on payoff relevant state variables.

Both the full-commitment and discretion approaches are to some extent unrealistic. Commitment does not match the observation that governments and other institutions have defaulted on past promises. Discretion rules out the possibility that governments achieve the benefits of making and keeping a promise, despite the *ex-post* incentive to renege. Roberds (1987) developed an approach – recently extended by Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010) – which escapes the “commitment vs discretion” dichotomy. Policymakers are endowed with a commitment technology, but with some exogenous and common knowledge probability they may succumb to the temptation to revise their plans. This approach has been labeled *quasi-commitment* or *loose commitment*.

Several questions can be addressed with the loose commitment approach. What are the gains of achieving more credibility? How does the possibility of future re-optimizations affect current outcomes and promises? What are the consequences of revising policy plans? How do occasional re-optimizations affect the shock propagation, volatilities, and cross-correlations between relevant variables? To answer these questions and derive the associated positive and normative implications, one must depart from the frameworks of commitment and discretion and consider instead loose commitment.

Nevertheless, due to some technical difficulties, the loose commitment approach has so far been limited to relatively simple and stylized models. The goal of this paper is to overcome this limitation. We propose a simple and relatively general algorithm to solve for the optimal policy plan under loose commitment in medium- and large-scale models typically used for monetary policy analysis. We show how these types of problems reduce to solving a system of linear difference equations, and do not present any additional challenge with respect to the commitment or discretion cases.

Our framework allows us not only to address the questions posed in complex monetary policy models, but also to pose new questions and examine how additional economic features interact with imperfect commitment. For instance, central banks often and carefully devise communication strategies where future actions may be revealed to the public. In one of our applications we distinguish the shocks that require more commitment and may call for a more detailed planning and communication strategy.

Assuming plans' revisions to be stochastic events, rather than endogenous decisions, is clearly a simplification analogous in spirit to the Calvo pricing model. While more complex credibility settings can be easily imagined (e.g. an endogenous timing of re-optimizations), such complexity may become prohibitive in medium- and large-scale models. In those type of models, the tractable though simplified approach employed here is particularly valuable.

This paper is related to the literature on optimal monetary policy in linear quadratic frameworks. Solution algorithms for full-commitment, together with a discussion about the computational aspects, have been developed by Currie and Levine (1993) and Söderlind (1999), among others. Methods to solve for (Markov-perfect) time-consistent equilibria are described in Backus and Driffill (1985), Söderlind (1999), and Dennis (2007). The main contribution of our paper is to extend these methodologies to address problems under loose commitment. To illustrate the benefits of our approach, the methodology is then applied to analyze the effects of commitment in the medium-scale model of Smets and Wouters (2007), which has arguably become one of the benchmark models in the dynamic stochastic general equilibrium literature.[1]

The paper continues as follows. In section 2 we introduce the general formulation of the model. In section 3 we study the optimal policy problem and describe the solution algorithm. Section 4 discusses the role of commitment in the Smets and Wouters (2007) model and section 5 concludes. We provide as supplementary material a collection of codes and documentation that implement our algorithm in a variety of models.

2 General form of the models

Consider a general linear model, whose structural equations can be cast in the form

$$A_{-1}y_{t-1} + A_0 y_t + A_1 E_t y_{t+1} + B v_t = 0, \qquad \forall t \tag{1}$$

where y_t indicates a vector of endogenous variables and v_t is a vector of serially uncorrelated exogenous disturbances with zero mean and $E v_t v_t' = \Sigma_v$. The vast majority of the models used for monetary policy analysis can be mapped into such formulation.

The common approach in the monetary policy literature is to assume that central banks have a quadratic loss function

$$\sum_{t=0}^{\infty} \beta^t y_t' W y_t. \tag{2}$$

In some cases, a purely quadratic objective function is consistent with a second-order approximation of a general time-separable utility function around an efficient steady-state (see e.g. Woodford (2003a)).[2] Moreover, quadratic loss functions have been shown to realistically describe central bank's behavior, even if they do not necessarily reflect the preferences of the underlying society.[3] In fact, and following Rogoff (1985), appointing a central banker who is more averse towards inflation than the overall public may be desirable in the limited commitment settings considered here.

Throughout the analysis we therefore maintain the assumption that the central bank's loss function is purely quadratic and may or may not reflect social preferences. Besides obvious tractability considerations, this feature guarantees that our methodology is flexible and directly applicable to most of the models used for monetary policy analysis.[4]

3 Optimal policy under loose commitment

In a loose commitment setting it is assumed that policymakers have access to a commitment technology but may occasionally revise their plans. More formally, suppose that the occurrence of a re-optimization is driven by a two-state Markov stochastic process

$$\eta_t = \begin{cases} 1 & \text{with Prob. } \gamma \\ 0 & \text{with Prob. } 1-\gamma. \end{cases} \tag{3}$$

At any given point in time if $\eta_t = 1$, previous commitments are honored. This event occurs with probability $0 \leq \gamma \leq 1$. Instead, if $\eta_t = 0$, previous promises are reneged and a new policy plan is formulated. This formulation nests both the full-commitment and discretion approaches as limiting cases where $\gamma = 1$ and $\gamma = 0$, respectively. More importantly, this formulation also spans the continuum between those two extremes.

Considering stochastic re-optimizations is a necessary simplification to address large scale models. Such an assumption also seems justified if the timing of plans revisions can be uncorrelated with the state of the economy. One possible candidate for such events is a change in the dominating view within a central bank due to time-varying composition of its decision-making committee. Another candidate is outside pressures of varying intensity exerted by politicians and the financial industry.[5] Alternatively, our approach can be interpreted as the reduced form of a model in which commitment to a policy is sustained by the threat of a punishment in case of re-optimization. If the punishment requires *a priori* coordination among private agents and in some random periods cannot be implemented, then such a model may bear similarities with our approach.[6] These are reasons for which our model can bear similarities with one where the re-optimization decision is endogenous. Whether our approach is plausible from an empirical perspective would require an estimation exercise. In later sections we do contrast our model with the data.

Following Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010), the policymaker's problem can be written as

$$y_{-1}' P y_{-1} + d = \min_{\{y_t\}_{t=0}^{\infty}} E_{-1} \sum_{t=0}^{\infty} (\beta\gamma)^t \left[y_t' W y_t + \beta (1-\gamma) (y_t' P y_t + d) \right] \tag{4}$$

$$\text{s.t.} \quad A_{-1} y_{t-1} + A_0 y_t + \gamma A_1 E_t y_{t+1} + (1-\gamma) A_1 E_t y_{t+1}^r + B v_t = 0. \quad \forall t \geq 0$$

The terms $y_{t-1}' P y_{t-1} + d$ summarize the value function at time t when a re-optimization occurs ($\eta_t = 0$). Since the problem is linear quadratic, the value function is quadratic and summarized by the state variables y_{t-1} and a term reflecting the stochastic nature of the problem. The matrix P and the scalar d have to be obtained in the solution procedure as shown in later sections.[7] The appendix discusses additional details related to the problem defined above.

The objective function is given by an infinite sum discounted at the rate $\beta\gamma$ summarizing the history in which re-optimizations never occur. Each term in the summation is composed of two parts. The first part is the period loss function. The second part indicates the value the policymaker obtains if a re-optimization occurs in the next period.

The policymaker faces a sequence of constraints, where in any period t expectations of future variables are an average between two terms. The first term (y_{t+1}), with weight γ, relates to the allocations prevailing when current plans are honored. The second term y^r_{t+1}, with weight $(1-\gamma)$, refers to the choices made in period $t+1$ if a re-optimization occurs (i.e. if $\eta_{t+1}=0$). As in the Markov-perfect literature, we assume that expectations about choices following a re-optimization only depend on state-variables.

$$E_t y^r_{t+1} = \tilde{H} y_t. \tag{5}$$

The policymaker cannot decide directly on the allocations implemented if a re-optimization occurs and therefore the matrix $\tilde{H}$ is taken as given.

For any $\tilde{H}$, the policymaker's problem can be solved using recursive methods. We follow the approach of Kydland and Prescott (1980) and Marcet and Marimon (2009), and write the Lagrangean associated with the optimal policy problem

$$\begin{aligned} \mathcal{L} \equiv E_{-1} \sum_{t=0}^{\infty} (\beta\gamma)^t \Big\{ & y_t' \left[W + (1-\gamma)\beta P \right] y_t + \lambda_{t-1}' \beta^{-1} A_1 y_t + \\ & \lambda_t' \left[A_{-1} y_{t-1} + \left(A_0 + (1-\gamma) A_1 \tilde{H} \right) y_t + B v_t \right] \Big\} \\ & \lambda_{-1} = 0 \\ & \tilde{H}, y_{-1} \text{ given.} \end{aligned} \tag{6}$$

This Lagrangean can be written recursively by expanding the state of the economy to include the Lagrange multiplier vector λ_{t-1}. The solution to the problem is then characterized by a time-invariant policy function

$$\begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} H_{yy} & H_{y\lambda} \\ H_{\lambda y} & H_{\lambda\lambda} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \begin{bmatrix} G_y \\ G_\lambda \end{bmatrix} v_t, \tag{7}$$

where the matrices H and G depend on the unknown matrix $\tilde{H}$.

When a re-optimization occurs in a given period t, the vector λ_{t-1} must be reset to zero. This result, formally proved by Debortoli and Nunes (2010), has an intuitive interpretation. A re-optimization implies that all the past promises regarding current and future variables are no longer binding.

According to equation (7) and setting $\lambda_{t-1} = 0$, it follows that $y_t^r = H_{yy}y_{t-1} + G_y v_t$. Moving this equation forward one period and taking expectations, one obtains $E_t y_{t+1}^r = H_{yy}y_t$. For this expression to be consistent with equation (5), it must be that in a rational expectations equilibrium

$$H_{yy} = \tilde{H}. \tag{8}$$

Given our formulation, the optimal policy under loose commitment can be found as the solution of a fixed point problem in the matrix H. In what follows we propose an algorithm to solve for that fixed point.

3.1 Solution algorithm

We start by writing the first-order conditions of the Lagrangean (6):

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = \left[A_0 + (1-\gamma) A_1 H_{yy}\right] y_t + \gamma A_1 E_t y_{t+1} + A_{-1} y_{t-1} + B v_t = 0 \tag{9}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial y_t} &= 2W y_t + \beta (1-\gamma) A_{-1}' E_t \lambda_{t+1}^r + \left[A_0 + (1-\gamma) A_1 H_{yy}\right]' \lambda_t \\ &+ \mathcal{I}_\gamma \beta^{-1} A_1' \lambda_{t-1} + \beta\gamma A_{-1}' E_t \lambda_{t+1} = 0. \end{aligned} \tag{10}$$

The vector equation (9) corresponds to the structural equation (1), where we have used equations (5) and (8) to substitute for the term $E_t y_{t+1}^r$. As a result, the unknown matrix H_{yy} enters equation (9). That matrix also enters equation (10), reflecting that y_t can be used to affect the expectations of y_{t+1}^r. The term λ_{t+1}^r in equation (10) constitutes the derivative of the value function w.r.t. y_t. This derivative can be obtained using the envelope condition

$$\frac{\partial y_t' P y_t}{\partial y_t} = 2P y_t = A_{-1}' E_t \lambda_{t+1}^r. \tag{11}$$

Finally, the term $\mathcal{I}_\gamma$ in equation (10) is an indicator function

$$\mathcal{I}_\gamma = \begin{cases} 0, \text{ if } \gamma = 0 \\ 1, \text{ otherwise} \end{cases} \tag{12}$$

and is used for convenience so that equation (10) is also valid under discretion ($\gamma = 0$), where the term $\beta^{-1} A_1' \lambda_{t-1}$ would not appear.[8]

There are many methods to solve linear rational expectation systems like (9)-(10), and standard routines are widely available (e.g. Sims (2002), Klein (2000), Collard and Juillard (2001)). Our computational implementation is based on the method of undetermined coefficients.

For a given guess of the matrix H, the law of motion (7) can be used to compute the expectations terms

$$E_t y_{t+1} = H_{yy} y_t + H_{y\lambda} \lambda_t \tag{13}$$

$$E_t \lambda_{t+1} = H_{\lambda y} y_t + H_{\lambda\lambda} \lambda_t \tag{14}$$

$$E_t \lambda^r_{t+1} = H_{\lambda y} y_t, \tag{15}$$

where the last equation follows from resetting the Lagrange multiplier λ_t to zero due to the re-optimization at $t+1$. Substituting these formulas into (9)-(10) one obtains

$$\Gamma_0 \begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} + \Gamma_1 \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \Gamma_v v_t = 0, \tag{16}$$

with

$$\Gamma_0 \equiv \begin{bmatrix} A_0 + A_1 H_{yy} & \gamma A_1 H_{y\lambda} \\ 2W + \beta A'_{-1} H_{\lambda y} & A'_0 + (1-\gamma) H'_{yy} A'_1 + \beta\gamma A'_{-1} H_{\lambda\lambda} \end{bmatrix}$$

$$\Gamma_1 \equiv \begin{bmatrix} A_{-1} & 0 \\ 0 & \beta^{-1} \mathcal{I}_\gamma A'_1 \end{bmatrix}, \qquad \Gamma_v \equiv \begin{bmatrix} B \\ 0 \end{bmatrix}.$$

The resulting law of motion is

$$\begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} = -\Gamma_0^{-1} \Gamma_1 \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} - \Gamma_0^{-1} \Gamma_v v_t, \tag{17}$$

where we are assuming the matrix Γ_0 to be non-singular.

The final step consists in verifying that this law of motion coincides with the initial guess, i.e. $H = -\Gamma_0^{-1}\Gamma_1$. If not, the guess-and-verify procedure is repeated until convergence. In summary, the algorithm proceeds as follows:

1. Using a guess H_{guess}, form Γ_0 and Γ_1.
2. Compute $H = -\Gamma_0^{-1}\Gamma_1$.

3. Check if $||H - H_{guess}|| < \xi$, where $||.||$ is a distance measure and $\xi > 0$. If the guess and the solution have converged, proceed to step 4. Otherwise, update the guess as $H_{guess} = H$ and repeat steps 1-3 until convergence.

4. Finally, form Γ_v and compute $G = -\Gamma_0^{-1}\Gamma_v$.

Clearly, there are many alternative algorithms to the one proposed. For example, for a given H the system of equations (9)-(10) could be solved using a generalized Schur decomposition as in Blanchard and Kahn (1980) or solving a quadratic matrix equation as in Uhlig (1995). For this reason, the non-singularity of the matrix Γ_0 is not essential. Also, the solution of the fixed point problem on the matrix H could be performed using a Newton-type method. Nevertheless, the procedure described above proved to be computationally more efficient.

The main message of our analysis is that solving for an optimal policy problem under loose commitment only requires solving a fixed point problem, which in a linear-quadratic framework is as simple as solving a system of linear equations. In addition, a loose commitment approach nests the full-commitment and discretion cases.

There are other practical advantages. For instance, Blake and Kirsanova (2010) show that some linear-quadratic models may display multiple equilibria under discretion. Those models may thus also exhibit multiple equilibria for intermediate commitment settings, depending on the initial guess for H. The advantage of our loose commitment approach, as implemented in the companion toolkit, is that there is a natural initial guess: the full-commitment solution, which is typically unique. The probability of commitment is then gradually reduced from full-commitment to discretion, using as guess the solution from the previous iteration.

In these iterations, the gradual reductions from $\gamma = 1$ to $\gamma = 0$ can be arbitrarily small, and this procedure can be viewed as a potential selection device among multiple discretionary equilibria.[9] Finally, even though multiple equilibria are a theoretical possibility, we found a unique solution in all the applications considered.[10]

3.2 Simulations and impulse responses

Once the matrices H and G have been obtained, it is straightforward to simulate the model for different realizations of the shocks and compute second moments and impulse response functions. For given initial conditions y_{-1}, λ_{-1}, and histories of the shocks $\{v_t, \eta_t\}_{t=0}^{T}$, the model simulation follows the formula

$$\begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} = H \begin{bmatrix} y_{t-1} \\ \eta_t \lambda_{t-1} \end{bmatrix} + G v_t. \tag{18}$$

The peculiarity of the loose commitment setting is that a history of the shock driving the re-optimizations (η_t) should also be specificied. Whenever $\eta_t = 0$, the Lagrange multiplier λ_{t-1} is reset to zero.

3.3 Welfare

For any initial condition $\begin{bmatrix} y'_{t-1} & \lambda'_{t-1} \end{bmatrix}$ the welfare measure, unconditional on the first realization of v_0, is given by

$$\begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix}' \widehat{P} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + d. \tag{19}$$

The matrix $\widehat{P}$ can be obtained by taking the derivative of the recursive formulation of the Lagrangean (6), thus obtaining

$$\widehat{P} = \frac{1}{2} \begin{bmatrix} 0 & A'_{-1} \\ \beta^{-1} A_1 & 0 \end{bmatrix} H. \tag{20}$$

Notice that in the most pertinent case with initial conditions $\lambda_{t-1} = 0$ the only relevant term would be the upper left block of $\widehat{P}$, which equals $A'_{-1} H_{\lambda y}$.

The constant d is given by

$$d = \frac{1}{1-\beta} tr \left[\Sigma_v \left(G' \widetilde{V} G + G' \begin{bmatrix} 0 \\ B \end{bmatrix} \right) \right] \tag{21}$$

with

$$\widetilde{V} = \begin{bmatrix} W & 0 \\ A_0 + (1-\gamma) A_1 H_{yy} & 0 \end{bmatrix} + \beta (1-\gamma) \begin{bmatrix} A'_{-1} H_{\lambda y} & 0 \\ 0 & 0 \end{bmatrix} + \beta \gamma \widehat{P}.^{11} \tag{22}$$

Alternatively, one can compute welfare conditional on the first realization of the shock, which is defined as follows:

$$\begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \\ v_t \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \\ v_t \end{bmatrix} + \widetilde{d} = y_t' W y_t \tag{23}$$
$$+ \beta\gamma E_t \left(\begin{bmatrix} y_t \\ \lambda_t \\ v_{t+1} \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_t \\ \lambda_t \\ v_{t+1} \end{bmatrix} + \widetilde{d} \right)$$
$$+ \beta (1-\gamma) E_t \left(\begin{bmatrix} y_t \\ 0 \\ v_{t+1} \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_t \\ 0 \\ v_{t+1} \end{bmatrix} + \widetilde{d} \right)$$

By definition of conditional welfare, it must be that

$$E_t \left(\begin{bmatrix} y_t \\ \lambda_t \\ v_{t+1} \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_t \\ \lambda_t \\ v_{t+1} \end{bmatrix} + \widetilde{d} \right) = \left(\begin{bmatrix} y_t \\ \lambda_t \end{bmatrix}' \widehat{P} \begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} + d \right), \tag{24}$$

and equation (23) can be rewritten as

$$\begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \\ v_t \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \\ v_t \end{bmatrix} + \widetilde{d} = \tag{25}$$
$$\left(H \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + G v_t \right)' \widetilde{V} \left(H \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + G v_t \right) +$$
$$\left(H \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + G v_t \right)' \left(\begin{bmatrix} 0 & \beta^{-1} A_1' \\ A_{-1} & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} v_t \right) + \beta d.$$

We can thus obtain the conditional welfare, for any given initial condition, by just evaluating the right-hand side of this last expression.

In these derivations we have computed welfare using the recursive formulation of the Lagrangean (6). As mentioned earlier, that formulation is equivalent to the original problem (4) only after imposing the initial condition $\lambda_{-1} = 0$. If one wants to evaluate the welfare according to the original formulation of equation (2), but for a different value of λ_{-1}, one needs to subtract $\lambda_{-1}\beta^{-1}A_1 E_{-1} y_0$ and $\lambda_{-1}\beta^{-1}A_1 y_0$ from equations (19) and (25), respectively.[12]

4 Application: a medium-scale closed economy model

In this section, we apply our methodology to the Smets and Wouters (2007) model. Needless to say, our purpose is neither to match business cycle properties nor to test the empirical plausibility of alternative commitment settings. We instead focus on examining the role of commitment in this benchmark medium-scale model.

The model includes nominal frictions in the form of sticky price and wage settings allowing for backward inflation indexation. It also features real rigidities – habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. The dynamics are driven by six orthogonal shocks: total factor productivity, two shocks affecting the intertemporal margin (risk premium and investment-specific technology shocks), two shocks affecting the intratemporal margin (wage and price-markup shocks), and an exogenous government spending shock. The model equations are omitted here for brevity and all parameters are calibrated to the posterior mode as reported in Smets and Wouters (2007).

Unlike Smets and Wouters (2007), we do not consider a specific interest rate rule nor the associated monetary policy shock. Instead, we assume that the central bank solves an optimal policy problem. By doing so, we exemplify how the degree of commitment and the re-optimization shocks affect the behavior of the central bank. We are not dismissing interest rate rules either from a normative or a positive perspective. In fact, it is widely known that optimal policy plans can be implemented in a variety of ways including targeting rules and instrument rules, of which interest rate rules are a subcase.

We explore the implications of two purely quadratic loss functions commonly used in the literature. The benchmark formulation is given by

$$U_t^b = w_\pi \pi_t^2 + w_y y_t^2 + w_i^b (i_t - i_{t-1})^2, \tag{26}$$

where π_t, y_t, and i_t denote respectively price inflation, output-gap, and the nominal interest rate. The alternative specification takes the form

$$U_t^a = w_\pi \pi_t^2 + w_y y_t^2 + w_i^a i_t^2. \tag{27}$$

Following Woodford (2003b), we set the parameters $w_\pi = 1$, $w_y = 0.003$, $w_i^b = 0.0176$, and $w_i^a = 0.0048$. The plausibility of these formulations and of the corresponding calibration is discussed in the following sections, where we analyze the importance of commitment from different perspectives. As explained in Section 2, deriving and carrying out the analysis with a micro-founded utility function is an interesting approach but goes beyond the scope of this paper.[13] The type of loss functions considered in this paper are used widely in central banks (e.g. Norges Bank (2011)) and in the literature describing or characterizing central bank behavior (see e.g. Rogoff (1985), Svensson (1999), Dennis (2004), Ilbas (forthcoming)).

4.1 What are the gains from commitment?

In Figure A-1, we plot the conditional welfare gains obtained for different levels of credibility. The continuous line (left axis) standardize welfare by the total gains of changing credibility from discretion to full-commitment. This standardization has the advantage that any affine transformation of the central bank's objective function would leave this welfare measure unchanged.

As expected, higher credibility leads to higher welfare.[14] More importantly, the figure suggests that if a central bank has low credibility to start with, a partial enhancement of its credibility will not deliver much of the welfare gains that credibility can potentially offer. On the other hand, a central bank with high credibility should be especially cautious. It will face severe welfare losses if its credibility is deemed to have been minimally affected. These results contrast with those obtained by Schaumburg and Tambalotti (2007) using a more stylized monetary policy model.

****** Figure 1: Welfare goes about here ******

Figure A-1 also considers another welfare measure that is useful to gauge losses for the objective functions employed by central banks and is described, for instance, in Jensen (2002). This measure (m) is the permanent deviation in the inflation target that would leave the central bank indifferent between full-commitment and another credibility level γ,

$$E_{-1}\sum_{t=0}^{\infty}\beta^t\left[w_\pi\left(\pi_{t,\gamma=1}-m\right)^2+w_y\left(y_{t,\gamma=1}\right)^2+w_i^b(i_{t,\gamma=1}-i_{t-1,\gamma=1})^2\right]=$$
$$E_{-1}\sum_{t=0}^{\infty}\beta^t\left[w_\pi\pi_{t,\gamma}^2+w_y y_{t,\gamma}^2+w_i^b(i_{t,\gamma}-i_{t-1,\gamma})^2\right]. \qquad (28)$$

In other words, this measure plots the permanent increase or decrease in the inflation level relative to the target of zero that would leave the central bank indifferent between the two credibility cases. A complete loss of credibility would be equivalent to a permanent change in the inflation rate of around 0.47 percent.[15]

Credibility may also affect the relative contribution of inflation and output-gap volatilities to the overall welfare loss. A higher credibility level translates into better management of the policy trade-offs because forward guidance is more effective as a policy tool. Therefore one might conjecture that higher credibility would reduce the volatilities of all welfare relevant variables. Figure A-2 exemplifies that such a conjecture does not always hold. The figure shows that for a given relative weight in the objective function, a loss in credibility leads to a rise in inflation volatility but a reduction in output-gap volatility. The reason is that stabilizing inflation is the most important welfare objective. A central bank with high credibility can achieve a higher welfare by promising to stabilize inflation even if doing so implies more output-gap volatility.

****** Figure 2: Credibility and volatility goes about here ******

Figure A-2 also discriminates among the points in the policy frontiers associated with doubling or halving w_π or w_y relative to the baseline calibration. Even considering such extreme calibrations of the welfare function does not change the results qualitatively. The finding that a loss in credibility increases inflation volatility but reduces output-gap volatility holds for those extreme calibrations as well.

4.2 Loose commitment and simple interest rate rules

The optimal policy under loose commitment can be implemented through targeting rules or through an appropriately defined interest rate rule.[16] In DSGE monetary policy models it is instead common to adopt simple reduced-form interest rate rules to describe the central bank's behavior. Clearly, such behavior is affected by the degree of commitment γ. An open question is to see how changes in γ are captured by the parameters of a simple rule. To address this question, we perform a Monte-Carlo exercise taking our model as the pseudo-true data generating process but estimating the interest rate rule

$$i_t = \phi_i i_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \epsilon_t, \tag{29}$$

where ϵ_t is assumed to be i.i.d. and normally distributed.

As a clarification, this exercise does not aim at finding the coefficients ϕ_i, ϕ_π, ϕ_y that would maximize welfare, which implies commitment to a simple interest rate rule. That is also an interesting approach followed for instance in Levine et al. (2008a). Here, we generate data from the model for several degrees of commitment and, as an econometrician would do, estimate the coefficients ϕ_i, ϕ_π, ϕ_y.

Table 1 presents the regression results. The coefficient estimates are similar to those found using actual data. In most cases, the coefficient on output-gap is small (and in some cases not significant), the coefficient on inflation is plausible, and there is a considerable degree of interest rate smoothing.[17] Most of the motive for interest-rate smoothing comes from commitment. Commitment implies that past policies matter for current allocations, thus introducing history dependence.[18] As a result, when commitment is high, the estimated values of ϕ_i are high even under the alternative loss function, where *per se* there is no interest-rate smoothing motive. Overall, the coefficient ϕ_i is more plausible for relatively loose commitment settings rather than with full-commitment.

Simple interest rate rules have been widely adopted to study central bank behavior across different periods of time. In that respect, our exercise shows that a change in the interest rate parameters $(\phi_i, \phi_\pi, \phi_y)$ should not be necessarily interpreted as a change in the central bank's preferences. Even if preferences remain unaltered, the reduced form interest rate parameters may change because of a loss of credibility.

The simple rule (29) captures fairly well the interest rate behavior, as signaled by the high value of the R^2. The R^2 is plausible but lower at intermediate degrees of commitment. The reason is that re-optimizations imply a non-linear change in the policy setting that the linear regression is not capturing well. The re-optimization uncertainty vanishes with full-commitment or discretion, and therefore those two cases can be better described by a linear rule. Also, the R^2 is lower for the alternative specification of the loss function. In that case, the absence of an interest rate smoothing motive in the objective function causes the interest rate to change more abruptly when re-optimizations occur. This result suggests that our benchmark loss function is more consistent with available estimates of the central bank behavior.

****** Table 1: Interest rate regressions goes about here ******

4.3 Business cycle properties under loose commitment

We now analyze the effects of commitment on business cycle properties. To that end, the probability of commitment is set to $\gamma = .90$, implying that policy re-optimizations occur on average every 10 quarters. That specific value is the one that minimizes the (weighted) difference between the (benchmark) model and the data, with respect to the Taylor-rule coefficients reported in table 1, and the statistics summarized in table 2.[19]

Impulse responses to different shocks are reported in Figures A-3-A-5. The solid line considers the specific history where re-optimizations do not occur over the reported horizon ($\eta_t = 1, \forall t$). On impact, the sign of the responses does not change with the commitment assumption. However, for each of the shocks considered, after about 6 quarters the response of the nominal interest rate does not lie between full-commitment (dashed line) and discretion (dash-dotted line). These differences arise because of the uncertainty about future re-optimizations, a feature unique to loose commitment settings.

****** Figure 3: Impulse responses to a wage markup shock goes about here ******

For example, the interest rate response to a positive wage markup shock, shown in Figure A-3, peaks after about 10 quarters – as opposed to a negligible response at a similar horizon both under full-commitment and discretion. In turn, the output-gap response is more prolonged, while both price and wage inflation are close to the values prevailing under commitment. Intuitively, the promise of a deeper and longer recession dampens inflation expectations and helps achieve a higher welfare. When the central bank re-optimizes (line with crosses), it reneges upon past promises. It then reduces the interest rate, causing inflation to increase and the output-gap to become closer to target. The bottom right panel shows that the welfare gain of re-optimizing in a given quarter – a measure of the time-inconsistency at each moment in time – is maximum after roughly 9 quarters. The central bank is fulfilling the promise of a deep recession, which becomes especially costly at that time because inflation is already below target and the output-gap is at its lowest level.

Similar reasoning also applies to productivity and government spending shocks.[20] In response to the latter shocks – as well as to other demand-type shocks – the output-gap and the two measures of inflation are well stabilized.

This occurs regardless of the degree of commitment, and as long as the central bank sets its policy optimally. This suggests that commitment would not be very important if these shocks were the main sources of business cycle fluctuations.[21] Also, the time-inconsistency problem, measured by the gains from re-optimizations (bottom right panel), is much smaller in response to technology and government spending shocks than in response to wage markup shocks.

****** Figure 4: Impulse responses to a productivity shock goes about here ******

****** Figure 5: Impulse responses to a government spending shock goes about here ******

Table 2 shows how commitment affects the second moments for some relevant variables. The correlation of output with the two measures of inflation is positive under full-commitment and becomes negative at intermediate degrees of commitment. The reason is that under full-commitment output and inflation are positively correlated not only conditionally on demand shocks, but also conditionally on technology and markup shocks. In response to the latter shocks, output and inflation move in opposite directions on impact, but after about 5 quarters they comove. Instead, with loose commitment, especially if a re-optimization has occurred, inflation and output move in opposite directions for a longer horizon. As a result, the correlation between inflation and output conditional on non-demand shocks, as well as the unconditional counterpart, changes sign with even a small departure from the full-commitment assumption.[22]

Table 2 also shows that in the data the correlation between output and price inflation is mildly negative, whereas the correlation between output and wage inflation is mildly positive – a feature that the loose commitment model with $\gamma = 0.9$ matches quite well. In addition, the relative volatility of interest rates is also more plausible with limited commitment settings.

****** Table 2: Effects of loose commitment on second moments goes about here ******

Finally, loose commitment changes the relative contribution of alternative shocks to business cycle fluctuations, as summarized in Figure A-6. This pattern is mostly evident for interest rate fluctuations. Under full-commitment about 55% of the fluctuations can be attributed to demand shocks. A small loss of credibility ($\gamma = .9$) is enough for this proportion to drop dramatically to about 17%. The contribution of wage and price markup shocks increases from 43% to 72%. The reason is that the interest-rate response to a demand shock does not change much with the degree of commitment. Instead, in response to markup shocks the interest rate barely responds under commitment, while it increases and remains high for a long period in limited commitment settings. For almost all the other variables, when commitment is lower, price markup shocks lose importance and wage markup shocks become more relevant. Hence, the variance decompositions and the earlier plots measuring time-inconsistency suggest that commitment is particularly important to stabilize wage markup shocks.

****** Figure 6: Variance decomposition goes about here ******

In summary, loose commitment has important effects on price and wage inflation dynamics, and nominal interest rates – the main variables for which the central bank is responsible. The impulse responses to different shocks, as well as the interest rate volatility, is not necessarily in between full-commitment and discretion. Finally, small departures from full-commitment change the sign of the correlation between output and inflation. In addition, the relative contribution of wage markup shocks to business cycle fluctuations increases dramatically, especially for interest rates and inflation.

5 Conclusions

Imperfect commitment settings overcome the dichotomy between full-commitment and discretion. In practice, policymakers have some degree of commitment that is not perfect – in some cases they keep a previously formulated policy plan whereas in other cases they reformulate those plans. Recent proposals of imperfect commitment settings were restricted to relatively simple and stylized models.

The contribution of this paper is to propose a method and a toolkit that extends the applicability of loose commitment to medium- and large-scale linear quadratic models typically used in monetary policy. We exemplified the method in the Smets and Wouters (2007) model, where we posed a variety of questions that our method can address and would remain otherwise unanswered.

Our easy-to-use toolkit permits several modeling extensions. For instance, it would be interesting to incorporate financial frictions, commodity price shocks, unemployment dynamics, and determine the importance of commitment in those cases. Since the optimal policy under loose commitment is not the average of the polar cases of full-commitment and discretion, examining the policy response to such shocks would be interesting *per se* and shed light on recent economic developments. Also, considering alternative intermediate credibility settings is certainly desirable, but technical and computational complexity may become prohibitive to address the medium- and large-scale models considered here. On a different note, our methodology could be exploited to analyze the plausibility of alternative commitment settings through an appropriate estimation exercise. We plan to pursue these projects in the near future.

Notes

[1]We have also tested our methodology with bigger models used for monetary policy analysis, such as the Norwegian Economy Model (NEMO) of the Norges Bank.

[2]In the presence of steady-state distortions, a purely quadratic objective can be obtained using a simple linear combination of the structural equations approximated to a second-order. However, as shown by Debortoli and Nunes (2006), this requires imposing the so-called "timeless perspective" assumption, which contrasts with the loose commitment settings considered in this paper. For an alternative approach, see Schmitt-Grohe and Uribe (2005).

[3]See for example the empirical analysis of Dennis (2004) and Ilbas (forthcoming).

[4]In the companion code, models with more lags, leads, constants, and serially correlated shocks are automatically transformed to be consistent with the formulation in equations (1) and (2). Stochastic targets and preference shocks can also be incorporated by suitably expanding the vector y_t.

[5]In the case of the United States, the reserve bank presidents serve one-year terms as voting members of the FOMC on a rotating basis, except for the president of the New York Fed. Furthermore, substantial turnover among the reserve bank presidents and the members of the Board of Governors arises due to retirement and outside options. With the (up to) seven members of the Board of Governors being nominated by the U.S. President and confirmed by the U.S. Senate, the composition of views in the FOMC may be affected by the views of the political party in power at the time of the appointment. Chappell et al. (1993) and Berger and Woitek (2005) find evidence of such effects in the U.S. and Germany, respectively.

[6]Such a framework would build on the seminal contributions of Chari and Kehoe (1990), and Kehoe and Levine (1993). A related approach using a model of imperfect information is described in Sleet (2001). Most of these frameworks model the private sector as a representative household therefore avoiding the coordination problem.

[7]The functional form of the value function is discussed, for instance, in Ljungqvist and Sargent (2004) (Ch. 5). In the initial period, the policymaker does not have to fulfil any previous promise and such period is therefore equivalent to a re-optimization.

[8]The indicator function is only needed because when deriving equation (10) we have divided all terms by $(\beta\gamma)^t$, which can be done only if $\gamma \neq 0$.

[9]Dennis and Kirsanova (2010) propose alternative selection devices based on the concepts of robustness and learnability.

[10]A recent work by Himmels and Kirsanova (2011) considers a model with multiple discretionary equilibria, and shows that a minimal degree of commitment is enough to eliminate that multiplicity. The authors also propose a way to detect and compute multiple equilibria, which we view as a complement to our analysis.

[11]The associated derivations, which follow the steps in Ljungqvist and Sargent (2004) (Ch. 5), are omitted for brevity and are available upon request.

[12]Our sample codes incorporate these correction terms.

[13]The reader is referred to Benigno and Woodford (2005), Benigno and Woodfoord (2006), Levin et al. (2005), Levine et al. (2008a), and Levine et al. (2008b).

[14]Debortoli and Nunes (2010) formally proved that welfare is increasing in the probability of commitment. Also, as discussed there, the shape of the relative welfare gains change with the commitment metric. Here, we are considering and comparing results in the literature along the probability of commitment metric.

[15]Computing the same measure relative to the output-gap target yields the value of 8.52 percent. This value is larger because the weight on output-gap stabilization is rather small. If, as stated in some central bank treaties such as the ECB, the only central bank's goal is to stabilize inflation, then this value would be infinity. Also note that this value is unrelated to consumption equivalent gains computed with a micro-founded utility function.

[16]Evans and Honkapohja (2003) discuss how interest rate rules can imple-

ment the optimal policy plan, while targeting rules are discussed by Giannoni and Woodford (2010) in a general framework and by Debortoli and Nunes (2011) in a loose commitment setting.

[17]For comparability with some studies the coefficient on inflation and output-gap should be adjusted as $\phi_\pi/(1-\phi_i)$ and $\phi_y/(1-\phi_i)$, respectively.

[18]For example, an optimal policy plan under full-commitment displays history dependence even when all the disturbances are i.i.d. and in the absence of natural state variables. See e.g. Galí (2008, ch. 5).

[19]In particular, we chose the value of γ through the simulated method of moments (SMM), using the (inverse) of the estimated variance-covariance matrix of the statistics over the sample 1970:Q1 - 2008:Q3 as the weighting matrix. The resulting value of γ would be very similar if targeting only the Taylor-rule coefficients (.93), or with the alternative objective function (.94).

[20]The responses to other shocks also present the same features and are omitted for brevity, but are available upon request.

[21]However, this result is not obvious in the current model. The presence of both price and wage rigidities implies a trade-off between inflation and output stabilization, and thus a scope for commitment, even in response to demand and technology shocks.

[22]The conditional cross-correlations are omitted for brevity and are available upon request.

References

Backus, D., Driffill, J., 1985. Inflation and reputation. American Economic Review 75 (3), 530–38.

Benigno, P., Woodfoord, M., 2006. Linear-quadratic approximation of optimal policy problems. NBER Working Paper (12672).

Benigno, P., Woodford, M., December 2005. Inflation stabilization and welfare: The case of a distorted stady state. Journal of the European Economic Association 3 (6), 1185–1236.

Berger, H., Woitek, U., 2005. Does conservatism matter? a time-series approach to central bank behaviour. Economic Journal 115 (505), 745–766.

Blake, A. P., Kirsanova, T., 2010. Discretionary policy and multiple equilibria in LQ RE models. manuscript, University of Exeter.

Blanchard, O. J., Kahn, C. M., 1980. The solution of linear difference models under rational expectations. Econometrica 48 (5), 1305–1311.

Chappell, H., Havrilesky, T., McGregor, R., 1993. Partisan monetary policies: Presidential influence through the power of appointment. The Quarterly Journal of Economics 108 (1), 185–218.

Chari, V. V., Kehoe, P. J., 1990. Sustainable plans. Journal of Political Economy 98 (4), 783–802.

Collard, F., Juillard, M., June 2001. A higher-order taylor expansion approach to simulation of stochastic forward-looking models with an application to a nonlinear phillips curve model. Computational Economics 17 (2-3), 125–39.

Currie, D., Levine, P., 1993. Rules, Reputation and Macroeconomic Policy Coordination. UK Cambridge. Cambridge University Press.

Debortoli, D., Nunes, R., 2006. On linear-quadratic approximations. Universitat Pompeu Fabra. Manuscript.

Debortoli, D., Nunes, R., 2010. Fiscal policy under loose commitment. Journal of Economic Theory 145 (3), 1005–1032.

Debortoli, D., Nunes, R., 2011. Monetary regime switches and unstable objectives. International Finance Discussion Papers 1036.

Dennis, R., 2004. Inferring policy objectives from economic outcomes. Oxford Bulletin of Economics and Statistics 66 (1), 735–764.

Dennis, R., 2007. Optimal policy in rational expectations models: New solution algorithms. Macroeconomic Dynamics 11 (01), 31–55.

Dennis, R., Kirsanova, T., 2010. Expectations traps and coordination failures: selecting among multiple discretionary equilibria. Working Paper Series 2010-02, Federal Reserve Bank of San Francisco.

Evans, G. W., Honkapohja, S., 2003. Expectations and the stability problem for optimal monetary policies. Review of Economic Studies 70 (4), 807–824.

Galí, J., 2008. Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. Princeton, New Jersey. Princeton University Press.

Giannoni, M., Woodford, M., 2010. Optimal target criteria for stabilization policy. NBER Working Paper 15757.

Himmels, C., Kirsanova, T., 2011. Expectations traps and monetary policy with limited commitment. manuscript, University of Exeter.

Ilbas, P., forthcoming. Revealing the Preferences of the US Federal Reserve. Journal of Applied Econometrics.

Jensen, H., 2002. Targeting nominal income growth or inflation? American Economic Review 92 (4), 928–956.

Kehoe, T. J., Levine, D. K., 1993. Debt-constrained asset markets. Review of Economic Studies 60 (4), 865–88.

Klein, P., 2000. Using the generalized schur form to solve a multivariate linear rational expectations model. Journal of Economic Dynamics and Control 24 (10), 1405–1423.

Kydland, F. E., Prescott, E. C., 1977. Rules rather than discretion: The inconsistency of optimal plans. Journal of Political Economy 85 (3), 473–91.

Kydland, F. E., Prescott, E. C., 1980. Dynamic optimal taxation, rational expectations and optimal control. Journal of Economic Dynamics and Control 2(1), 79–91.

Levin, A., Onatski, A., Williams, J., Williams, N., 2005. Monetary policy under uncertainty in micro-founded macroeconometric models. In: Gertler, M., Rogoff, K. (Eds.), NBER Macroeconomics Annual 2005, Volume 20. Cambridge MA. MIT Press, pp. 229–312.

Levine, P., McAdam, P., Pearlman, J., 2008a. Quantifying and sustaining welfare gains from monetary commitment. Journal of Monetary Economics 55 (7), 1253 – 1276.

Levine, P., Pearlman, J., Pierse, R., 2008b. Linear-quadratic approximation, external habit and targeting rules. Journal of Economic Dynamics and Control 32 (10), 3315–3349.

Ljungqvist, L., Sargent, T. J., 2004. Recursive Macroeconomic Theory. Cambridge, MA. The MIT Press.

Marcet, A., Marimon, R., 2009. Recursive contracts. Universitat Pompeu Fabra. Working Paper.

Norges Bank, 2011. Norges Bank Monetary Policy Report, June. http://www.norges-bank.no, 1–46.

Roberds, W., 1987. Models of policy under stochastic replanning. International Economic Review 28 (3), 731–755.

Rogoff, K., 1985. The optimal degree if commitment to an intermediate monetary target. Quarterly Journal of Economics 100 (4), 1169–89.

Schaumburg, E., Tambalotti, A., 2007. An investigation of the gains from commitment in monetary policy. Journal of Monetary Economics 54 (2), 302–324.

Schmitt-Grohe, S., Uribe, M., 2005. Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In: Gertler, M., Rogoff, K. (Eds.), NBER Macroeconomics Annual 2005, Volume 20. Cambridge MA. MIT Press, pp. 383–462.

Sims, C., October 2002. Solving linear rational expectations models. Computational Economics 20, 1–20.

Sleet, C., 2001. On credible monetary policy and private government information. Journal of Economic Theory 99, 338–376.

Smets, F., Wouters, R., 2007. Shocks and frictions in us business cycles: A bayesian dsge approach. American Economic Review 97 (3), 586–606.

Söderlind, P., 1999. Solution and estimation of RE macromodels with optimal policy. European Economic Review 93, 813–823.

Svensson, L. E. O., 1999. Inflation targeting as a monetary policy rule. Journal of Monetary Economics 43 (3), 607–654.

Uhlig, H., 1995. A toolkit for analyzing nonlinear dynamic stochastic models easily. Institute for Empirical Macroeconomics Discussion Paper 101, Federal Reserve Bank of Minneapolis.

Woodford, M., 2003a. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton, New Jersey. Princeton University Press.

Woodford, M., 2003b. Optimal interest-rate smoothing. Review of Economic Studies 70 (4), 861–886.

A-1 Appendix

The problem of the central bank under full commitment is:

$$V_0 = \min_{\{y_t\}_{t=0}^{\infty}} E_{-1} \sum_{t=0}^{\infty} \beta^t y_t' W y_t \tag{A-1}$$
$$\text{s.t.} \quad A_{-1}y_{t-1} + A_0 y_t + A_1 E_t y_{t+1} + B v_t = 0. \quad \forall t \geq 0$$

where V_0 is the value function obtained at time 0. Treating the vector y_t as state-variables and noting that the value function is quadratic, one obtains $V_0 = y_{-1}' P y_{-1} + d$ where the matrix P and the constant d need to be determined in the equilibrium solution.

The problem under limited commitment needs to be adapted because the central bank can only choose directly the allocations corresponding to histories where it retains commitment. If the commitment technology is broken in a certain time period, previous decisions are disregarded and policy is reoptimized – such an event is analogous to a new central bank or chairman taking over. While the formulation (A-1) remains valid, before taking first order conditions, it is helpful to write explicitly the allocations upon which the central bank appointed in $t = 0$ is deciding.

The treatment of the constraints is easier and needs to be adapted according to

$$\begin{aligned} A_{-1}y_{t-1} + A_0 y_t + A_1 Prob(\eta_{t+1} = 1) E_t \left(y_{t+1} | \eta_{t+1} = 1\right) \\ + A_1 Prob(\eta_{t+1} = 0) E_t \left(y_{t+1}^r | \eta_{t+1} = 0\right) + B v_t = 0, \end{aligned} \tag{A-2}$$

where y_{t+1}^r refers to the allocations in case a re-optimization occurs. Given our assumptions on the distribution of the re-optimization shocks, the expression above is simplified to:

$$A_{-1}y_{t-1} + A_0 y_t + \gamma A_1 E_t y_{t+1} + (1-\gamma) A_1 E_t y_{t+1}^r + B v_t = 0, \tag{A-3}$$

where we simplified the notation on the expectations operator since we already distinguish y_{t+1} from y_{t+1}^r

The objective function also needs to be adapted using similar steps. Whenever a re-optimization occurs, a new central bank takes over and the current central bank cannot decide on those allocations directly. However, the allocations decided by the new central bank still provide utility for the current central bank. Such lifetime utility in case of re-optimization is conveniently summarized through a value function V^r. Writing a few terms of the central bank's objective function:

$$\begin{aligned}
&t = 0 : y_0' W y_0 \\
&t = 1 : +\beta \left[\gamma y_1' W y_1 + (1-\gamma) V_1^r\right] \\
&t = 2 : +\beta^2 \left[\gamma^2 y_2' W y_2 + \gamma (1-\gamma) V_2^r\right] \\
&t > 2 : +...
\end{aligned} \tag{A-4}$$

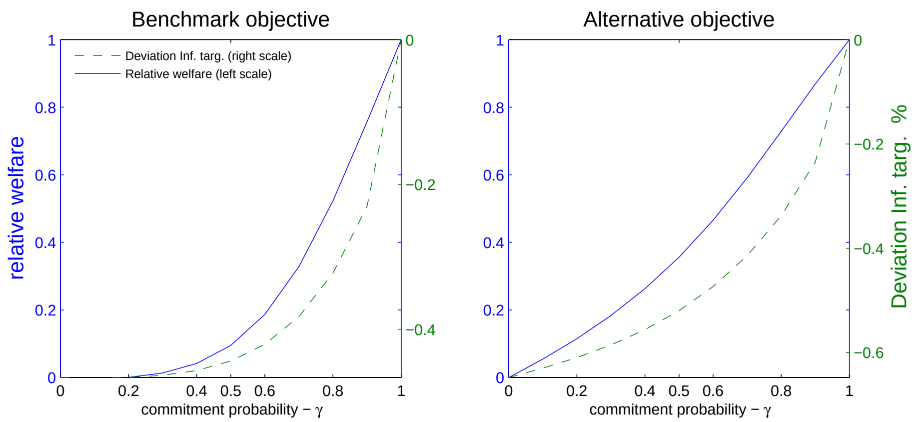
In period $t = 0$, the welfare terms are written explicitly because the current central bank decides directly on those. In period $t = 1$, discounted at rate β, the central bank chooses the allocations in case a re-optimization does not occur. Such event has probability γ. With probability $(1-\gamma)$ a re-optimization occurs and the lifetime utility from that node onwards is summarized by V_1^r. Period $t = 2$, as well as later periods follow the same logic. Grouping all those terms together yields:

$$\sum_{t=0}^{\infty} (\beta\gamma)^t \left[y_t' W y_t + \beta (1-\gamma) V_{t+1}^r\right]. \tag{A-5}$$

We solve for an equilibrium where the problem of the current and future central banks coincide. In the initial period $t = 0$, the central bank does not have to fulfil any previous promise and such period is therefore equivalent to a re-optimization. For these reasons, we obtain $V_{t+1}^r = y_t' P y_t + d$; the matrix P and scalar d are the same as before but now one considers the state variables for the corresponding period. Making the relevant substitutions, the planner's problem is therefore given by (4).

Several details of this formulation are available in Debortoli and Nunes (2010). In that paper, we show in detail that all the nodes of the possible tree of events are covered. We also show that, given the value functions V_{t+1}^r and the policy functions when a re-optimization occurs $\left(E_t y_{t+1}^r = \tilde{H} y_t\right)$, the problem is well posed and fits the framework of Marcet and Marimon (2009). As described in the main text, the solution procedure requires that V_{t+1}^r and $E_t y_{t+1}^r$ are consistent with the equilibrium (through the matrices P, $\tilde{H}$ and the scalar d).

Figure A-1: Welfare



Notes: The figure plots the welfare gains from commitment for the benchmark (left-panel) and the alternative (right-panel) objective function. The continuous line (left-scale) indicates the relative gains from full discretion to a degree of commitment γ, i.e. $(V_\gamma - V_{\gamma=0})/(V_{\gamma=1} - V_{\gamma=0})$. This measure corresponds to conditional welfare and the results are robust to unconditioning on the shocks. The dashed line (right-scale) indicates equivalent permanent deviation from the inflation target according to equation (28), i.e. $m^2 = (1-\beta)\left(V_{\gamma=1} - V_\gamma\right)/w_\pi$. We plot the negative of $-m$ for the convenience of plotting an increasing function in γ.

Table 1: Interest rate regressions

	Benchmark Loss Function 1	0.9	0.5	0	Alternative Loss Function 1	0.9	0.5	0	U.S. Data (1970-2008)
ϕ_π	0.241	0.207	1.204	1.914	0.175	0.057	0.725	2.334	0.128
	(0.047)	(0.103)	(0.141)	(0.048)	(0.043)	(0.138)	(0.312)	(0.072)	(0.039)
ϕ_y	0.002	-0.003	0.059	0.105	0.002	-0.010	-0.030	0.12	0.042
	(0.003)	(0.007)	(0.014)	(0.005)	(0.002)	(0.009)	(0.033)	(0.008)	(0.009)
ϕ_i	0.971	0.926	0.875	0.75	0.972	0.843	0.503	0.159	0.926
	(0.022)	(0.033)	(0.038)	(0.015)	(0.022)	(0.06)	(0.062)	(0.027)	(0.028)
R^2	0.923	0.865	0.843	0.977	0.921	0.759	0.416	0.930	0.947

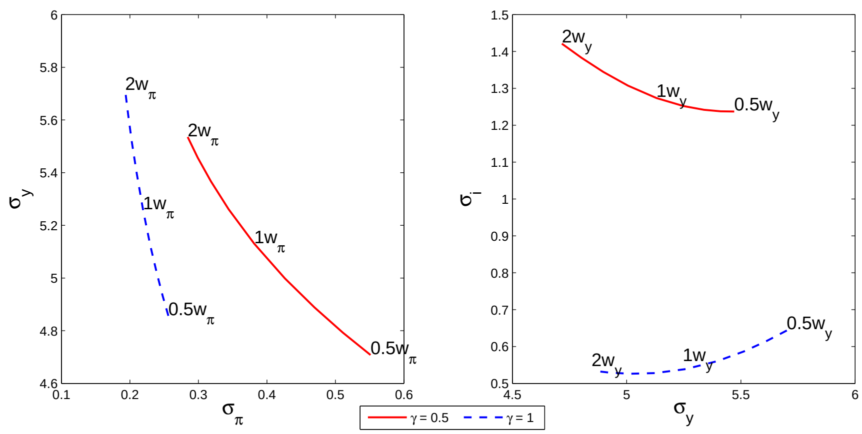
Notes: The table displays the coefficients and standard deviations corresponding to estimating equation (29) in the original model. The Monte-Carlo exercise is comprised of 1000 estimations of 200 periods each (roughly corresponding to the size of actual samples). The average standard deviations across simulations are reported in parenthesis. The last row displays the R^2. The panel on the left and the center correspond to the benchmark and alternative welfare functions, respectively. The sample regarding the U.S. data goes from 1970:Q1 until 2008:Q3, where the latest data is determined by the beginning of the zero lower bound period. The output-gap data corresponds to the CBO measure.

Table 2: Effects of loose commitment on second moments

	Model Full-Com.	Loose Com. $\gamma = 0.9$	Discr.	**U.S. Data** (1970 - 2008)
Standard deviation (w.r.t. output)				
Output-gap	0.83	0.84	0.83	0.74
Price inflation	0.04	0.04	0.07	0.21
Wage inflation	0.08	0.08	0.09	0.26
Interest rate	0.09	0.15	0.18	0.29
Cross-correlations with output				
Output-gap	0.87	0.88	0.86	0.90
Price inflation	0.05	-0.17	-0.70	-0.13
Wage inflation	0.21	0.13	-0.38	0.05
Interest rate	-0.34	-0.49	-0.56	-0.32

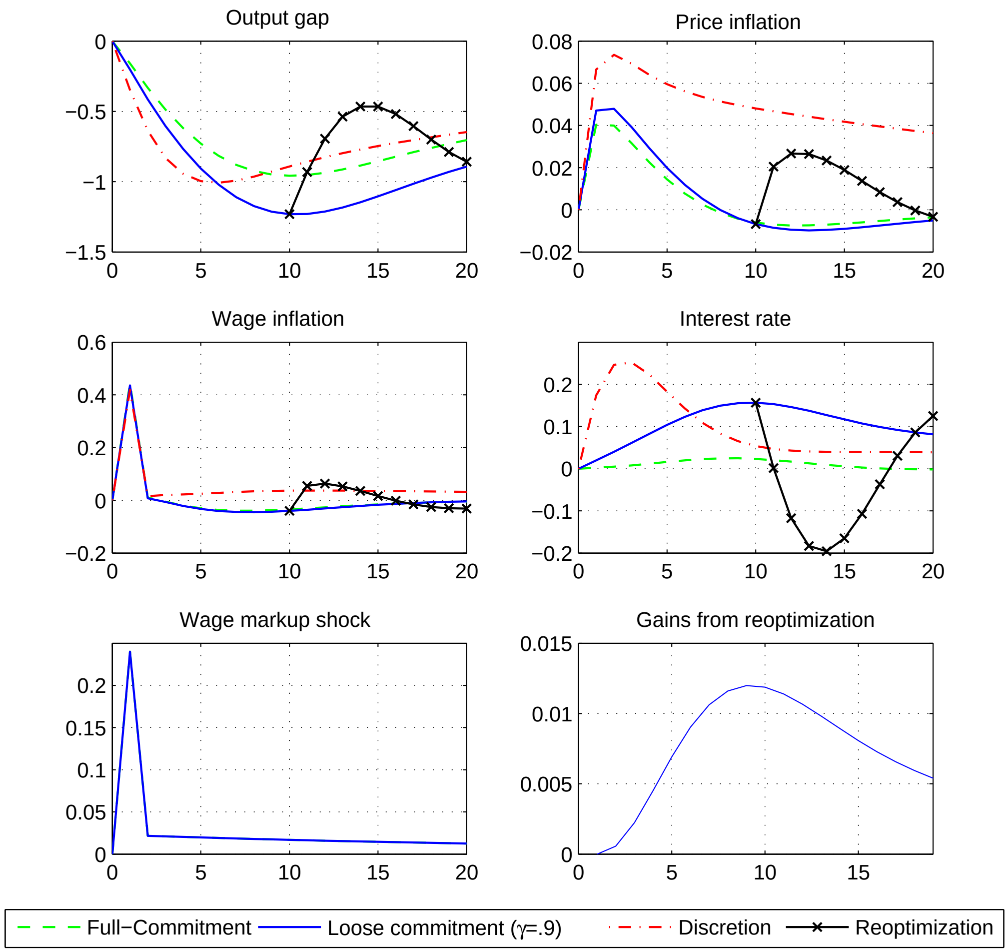
Notes: The table displays several statistics for the output-gap, inflation, wage inflation, and the interest rate. The model statistics are computed with 1000 simulations of 200 periods each. The sample regarding the U.S. data goes from 1970:Q1 until 2008:Q3, where the latest data is determined by the beginning of the zero lower bound period. The output-gap data corresponds to the CBO measure.

Figure A-2: Credibility and volatility



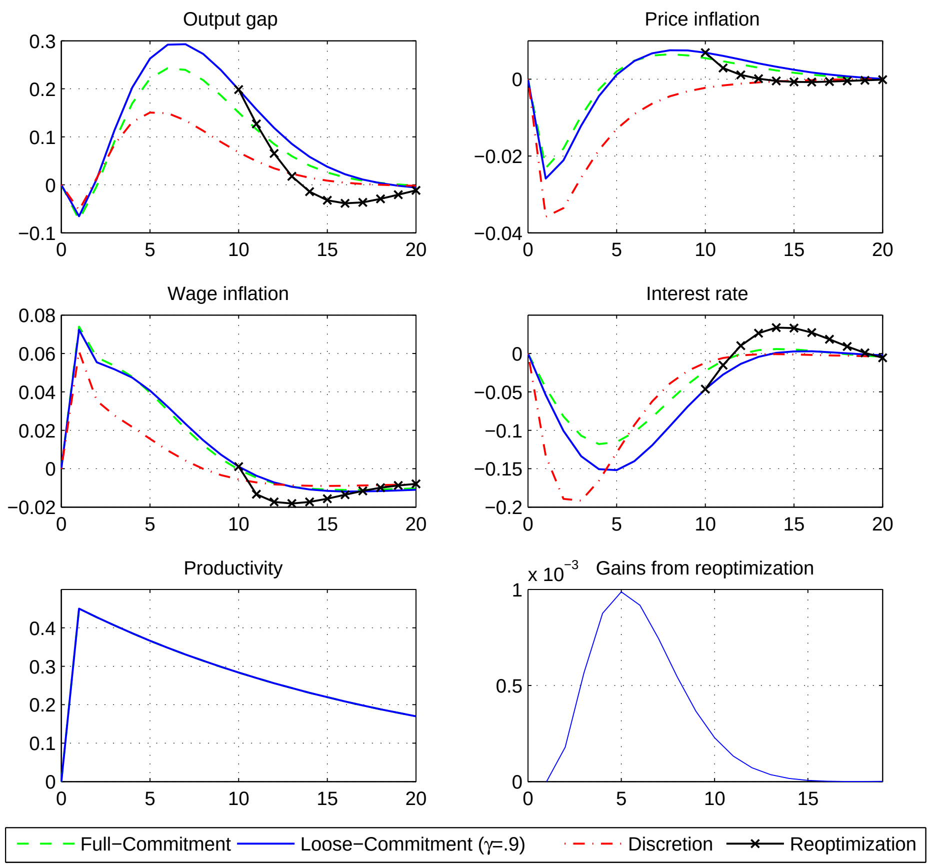
Notes: the figure plots the volatilities of inflation, output-gap, and interest rate for different credibility levels. The left and right panel change the weight on inflation and output-gap, respectively. The two panels plot several weights from half to double of the benchmark value. The solid and dashed lines consider the probability of commitment to be 0.5 and 1, respectively.

Figure A-3: Impulse responses to a wage markup shock



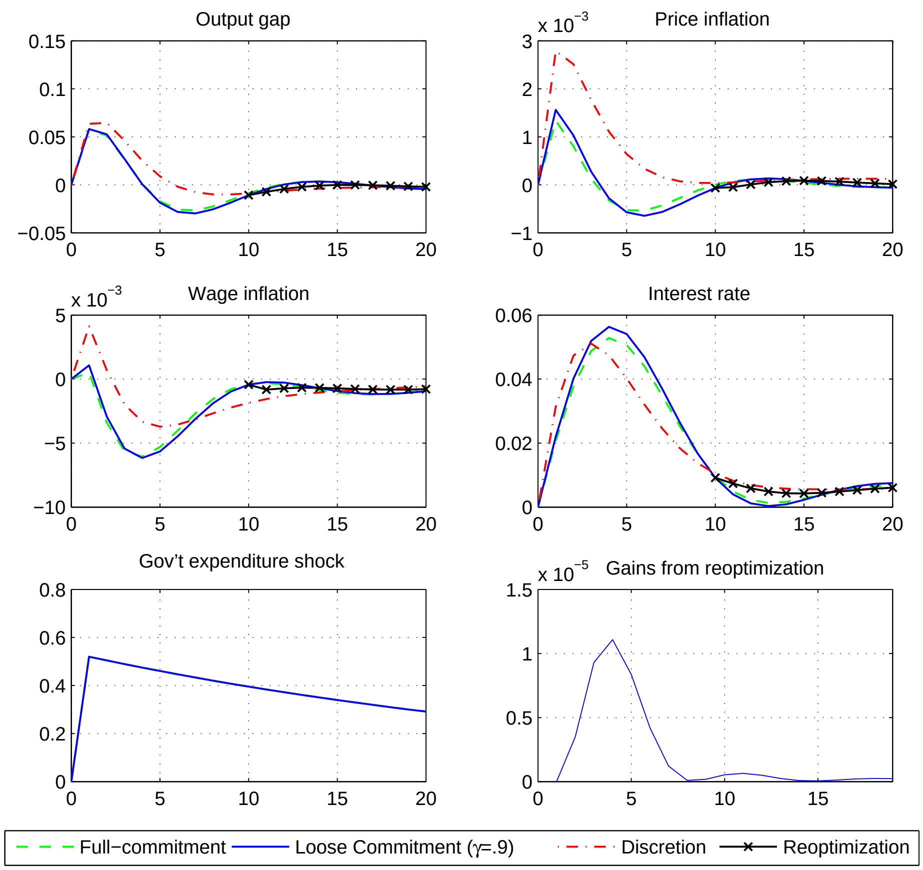
Notes: The figure plots the impulse responses to a one standard deviation shock, under different commitment settings. The solid line refers to a particular history where the probability of commitment $\gamma = .9$ and re-optimizations do not occur ($\eta_t = 1, \forall t$). The line with crosses refers to a particular history where the probability of commitment $\gamma = .9$ and a single re-optimization occurs after 10 quarters ($\eta_{10} = 0, \eta_t = 1, \forall t \neq 10$). For any quarter, the gains from re-optimization are computed as the welfare difference between keeping the announced plan vs reoptimizing in that particular quarter.

Figure A-4: Impulse responses to a productivity shock



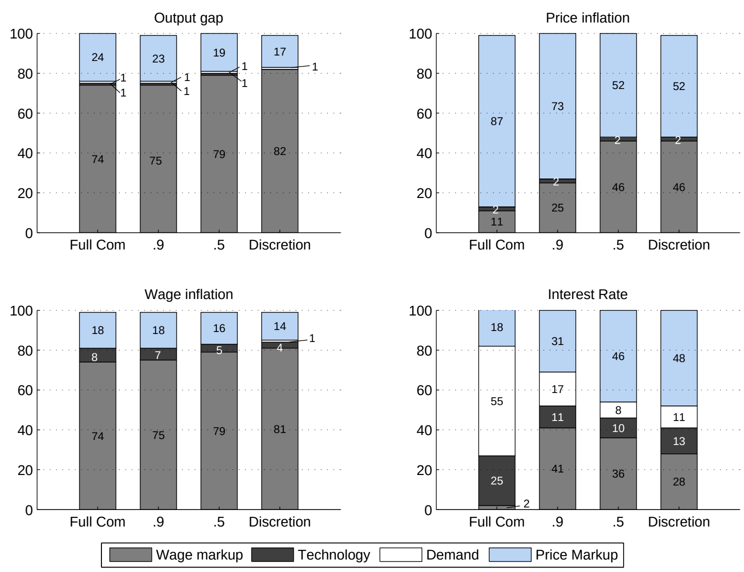
Notes: The figure plots the impulse responses to a one standard deviation shock, under different commitment settings. The solid line refers to a particular history where the probability of commitment $\gamma = .9$ and re-optimizations do not occur ($\eta_t = 1, \forall t$). The line with crosses refers to a particular history where the probability of commitment $\gamma = .9$ and a single re-optimization occurs after 10 quarters ($\eta_{10} = 0, \eta_t = 1, \forall t \neq 10$). For any quarter, the gains from re-optimization are computed as the welfare difference between keeping the announced plan vs reoptimizing in that particular quarter.

Figure A-5: Impulse responses to a government spending shock



Notes: The figure plots the impulse responses to a one standard deviation shock, under different commitment settings. The solid line refers to a particular history where the probability of commitment $\gamma = .9$ and re-optimizations do not occur ($\eta_t = 1, \forall t$). The line with crosses refers to a particular history where the probability of commitment $\gamma = .9$ and a single re-optimization occurs after 10 quarters ($\eta_{10} = 0, \eta_t = 1, \forall t \neq 10$). For any quarter, the gains from re-optimization are computed as the welfare difference between keeping the announced plan vs reoptimizing in that particular quarter.

Figure A-6: Variance decomposition



Notes: The figure displays the contribution of different shocks to the variance of our variables, under different commitment scenarios. For convenience, risk premium, investment specific, and government spending shocks have been grouped as "demand" shocks. The model statisitics are computed with 1000 simulations of 200 periods each.