An Advanced Course On The Science and Art of DSGE Modelling

Day 3: Classical Heterogeneous Agent Models

Szabolcs Deák

Kirill Shakhnov

Hyungseok Joo

September 2021

Model Economy

Model Economy: Firms

- An RBC model with incomplete markets.
- The representative firm combines labour, H_t, and capital services, K_{t-1}, into a final good, Y_t:

$$Y_t = F(A_t, H_t, K_{t-1}) = (A_t H_t)^{\alpha} K_{t-1}^{1-\alpha} \tag{1}$$

where A_t denotes labour productivity

- The first-order conditions:

$$W_t = \alpha \frac{Y_t}{H_t}, \tag{2}$$

$$R_t = (1-\alpha)\frac{Y_t}{K_{t-1}} + 1 - \delta \tag{3}$$

Model Economy: Households

- A unit mass of ex ante identical households.
- Household i faces an idiosyncratic risk to its labour income

$$\varepsilon_{i,t} h_{i,t} W_t$$

  where $\varepsilon_{i,t}$ is idiosyncratic labour productivity, $h_{i,t}$ is hours worked, and W_t is wage per efficiency unit common to all households.
- Households are endowed with one unit of time that is devoted entirely to labour, $h_{i,t} \equiv 1$.

Model Economy: Households

- Households can accumulate capital through savings:

$$k_{i,t} = R_t k_{i,t-1} + \varepsilon_{i,t} W_t - c_{i,t}$$

where $c_{i,t}$ and $k_{i,t}$ are consumption and capital holdings

- The maximization problem of household i is:

$$\max_{\{c_{i,t},k_{i,t}\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t U(c_{i,t}) \tag{4}$$

$$\text{s.t.} \quad c_{i,t} + k_{i,t} = R_t k_{i,t-1} + \varepsilon_{i,t} W_t \tag{5}$$

$$c_{i,t} \geq 0 \tag{6}$$

$$k_{i,t} \geq \underline{k} \tag{7}$$

- $\underline{k}$: a time- and state-independent lower bound on capital holdings (borrowing constraint)

Model Economy: Capital

- The role of capital in the model:
  - it is the only traded asset in the economy
  - its price is 1 (exchangeable for consumption one-to-one)
  - a store of value for the households to provide self-insurance against labour income risk: households are not permitted to trade with each other (incomplete market assumption)

Model Economy: Markets

- Market clearing requires:

$$H_t = \int \varepsilon_{i,t} h_{i,t} di \tag{8}$$

$$K_t = \int k_{i,t} di \tag{9}$$

- Since we assume $h_{i,t} \equiv 1$:

$$H_t = \int \varepsilon_{i,t} h_{i,t} di = \int \varepsilon_{i,t} di = E_t \varepsilon_{i,t} \tag{10}$$

- We constrain both aggregate and individual capital:

$$0 \leq K_t \qquad\qquad \underline{k} \leq k_{i,t}$$

Model Economy: Solution

- This model does not admit a representative household: markets are incomplete
- Representative household: a household whose maximization problem generates the relevant aggregate demand functions
- Aggregate demand will be a function of the wealth distribution.

Model Economy: Solution

- Consider an interior solution to the household's maximization problem:

$$U'(c_{i,t}) = \beta E_t\{U'(c_{i,t+1})R_{t+1}\} \tag{12}$$

- Using market clearing and the firm's FOC it becomes

$$U'(c_{i,t}) = \beta E_t \left\{ U'(c_{i,t+1}) \left[(1-\alpha) \left(\frac{A_{t+1}H_{t+1}}{\int k_{i,t} di} \right)^{-\alpha} + 1 - \delta \right] \right\} \tag{13}$$

- The wealth distribution at the end of period t is required to compute $c_{i,t}$ and consequently C_t.

Model Economy: Solution

- We can rewrite the households' maximization problem recursively:

$$V(k,\varepsilon,\lambda,A) = \max_{k'} U(Rk + \varepsilon W - k') + \beta E\{V(k',\varepsilon',\lambda',A')|\varepsilon,A\} \quad (14)$$

$$\text{s.t.} \quad \lambda' = \Gamma(\lambda, A, A') \quad (15)$$

$$k' \geq \underline{k} \quad (16)$$

- Variables without an $'$ are known at the beginning of the period.
- Variables with an $'$ are to be determined during the period.

Model Economy: Solution

- We can rewrite the households' maximization problem recursively:

$$V(k,\varepsilon,\lambda,A) = \max_{k'} U(Rk + \varepsilon W - k') + \beta E\{V(k',\varepsilon',\lambda',A')|\varepsilon,A\} \quad (14)$$

$$\text{s.t.} \quad \lambda' = \Gamma(\lambda, A, A') \quad (15)$$

$$k' \geq \underline{k} \quad (16)$$

- λ is the probability distribution of households over holdings of capital and labour productivity: it is a state variable

- Recall: households need to form expectations about λ' to determine their consumption.

Model Economy: Solution

- We can rewrite the households' maximization problem recursively:

$$V(k,\varepsilon,\lambda,A) = \max_{k'} U(Rk + \varepsilon W - k') + \beta E\{V(k',\varepsilon',\lambda',A')|\varepsilon,A\} \quad (14)$$

$$\text{s.t.} \quad \lambda' = \Gamma(\lambda, A, A') \quad (15)$$

$$k' \geq \underline{k} \quad (16)$$

- $\Gamma(\lambda, A, A')$: the distribution's law of motion (to be specified later)

- It allows households to form expectations about λ'.

Model Economy: Solution

- Suppose someone tells you the law of motion Γ. Is there a way to verify whether that law of motion is correct?
- We can use the market clearing conditions and the firm's FOCs to compute factor prices.
- Factor prices and the law of motion Γ allow us to solve the household's maximization problem to obtain individual decisions k'.
- Aggregating k' we can compute λ' and compare with the distribution implied by Γ itself.

A Generic Solution Algorithm

- Each algorithm we study follows the steps:

  1. Guess the law of motion Γ.
  2. Solve for the optimal individual decision rules given the law of motion.
  3. Aggregate the individual decisions in order to determine the aggregate demand functions.
  4. If the resulting aggregate demand functions are consistent with our guess about Γ, then we have found the solution.
  5. Otherwise update our guess. Rinse and repeat.