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- **Idiosyncratic households (after-tax) earnings process:**

$$\begin{aligned} \log Y_{it} &= \kappa_t + y_{it} = \kappa_t + z_{it} + \varepsilon_{it} \\ z_{it} &= z_{i,t-1} + \eta_{it} \end{aligned}$$

  - κ_t common deterministic experience profile
  - z_{it} permanent component, ε_{it} transitory component
  - z_{i0} is drawn from a given initial distribution

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- **Markets:** Households can borrow (up to $\underline{A} \leq 0$) and save through risk-free bond. Perfect annuity markets.
- **World interest rate**: r
- **Government:** Social security benefits $P(\mathbf{Y_i})$ paid to retirees
- **Budget constraints:**

$$C_{it} + A_{i,t+1} = (1+r)\,A_{it} + Y_{it}, \qquad \text{if } t < T^{ret}$$

$$C_{it} + \frac{\xi_t}{\xi_{t+1}} A_{i,t+1} = (1+r)\,A_{it} + P(\mathbf{Y_i}), \quad \text{if } t \geq T^{ret}$$

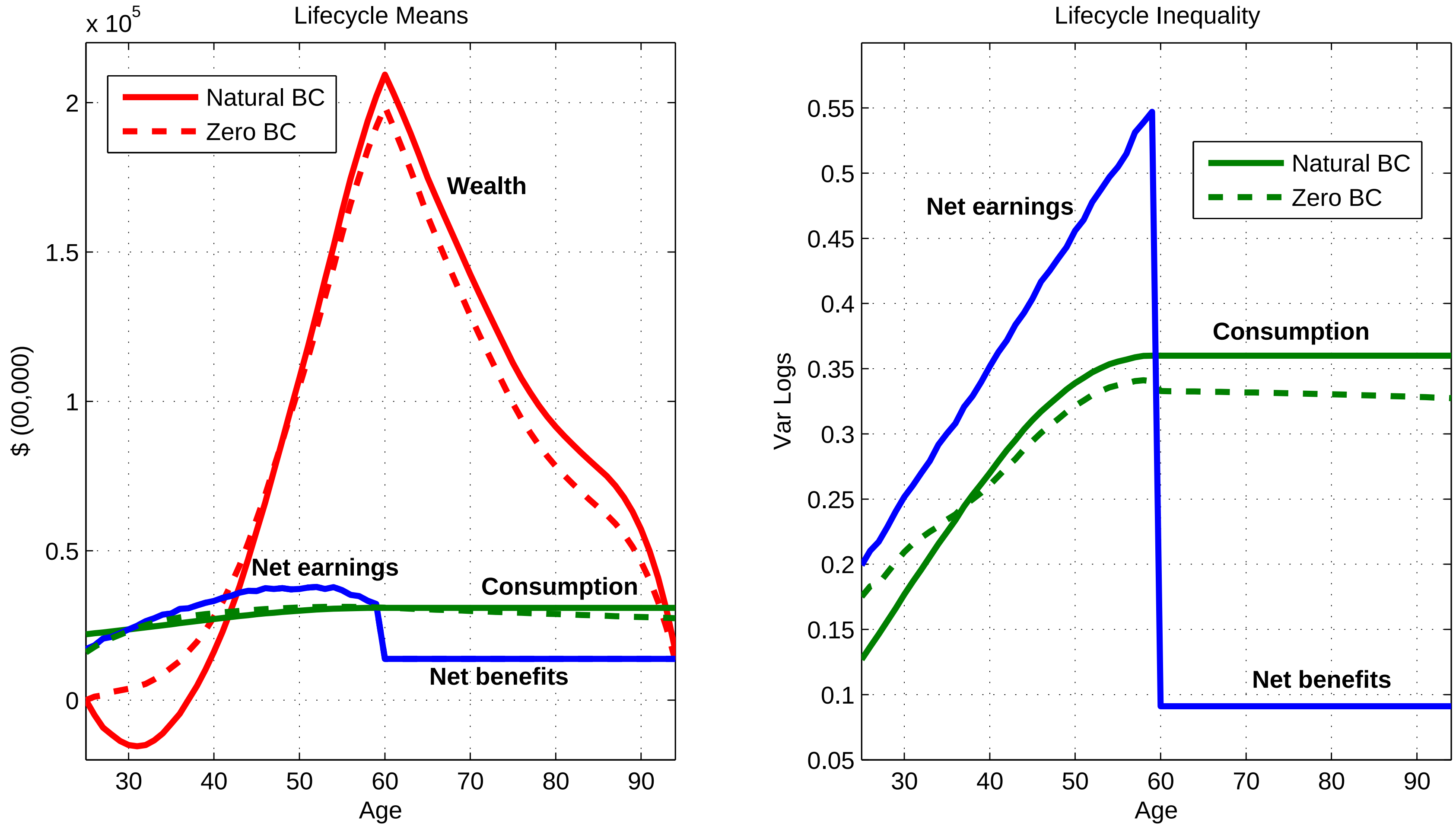
Calibration

- **Preferences:**
  - Relative risk aversion coefficient: $\gamma = 2$
  - Discount factor β to replicate aggregate net-worth-income ratio of 2.5 for bottom 95% of US households
- **Interest rate:** $r = 3\%$
- **Earnings process:**
  - Rise in earnings dispersion over lifecycle: $\sigma_\eta = 0.01$
  - Initial earnings dispersion: $\sigma_{z_0} = 0.15$
  - BPP estimate: $\sigma_\varepsilon = 0.05$

Calibration

- **Debt limit:** Natural or no-borrowing constraints
- **Initial wealth:** Zero or calibrated to net-worth distribution of 20-30 years-old
- **Social security:**
  1. Net earnings $\Rightarrow$ gross earnings by inverting Gouveia-Strauss tax function
  2. Benefits modelled as concave function of gross average lifetime earnings, as in US two-bendpoint system
  3. Benefits partially taxed

Lifecycle Implications



Baseline Economy

	Permanent Shock			Transitory Shock		
	Data BPP	Model BPP	Model TRUE	Data BPP	Model BPP	Model TRUE
Natural Borrowing Limit	0.36 (0.09)	0.22	0.24	0.95 (0.04)	0.94	0.94
Zero Borrowing Limit	0.36 (0.09)	0.08	0.24	0.95 (0.04)	0.82	0.82

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- Model has right amount of insurance wrt transitory shock (if borrowing limit is loose)

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- Model has right amount of insurance wrt transitory shock (if borrowing limit is loose)
- Model has less insurance than data wrt permanent shock

Baseline Economy

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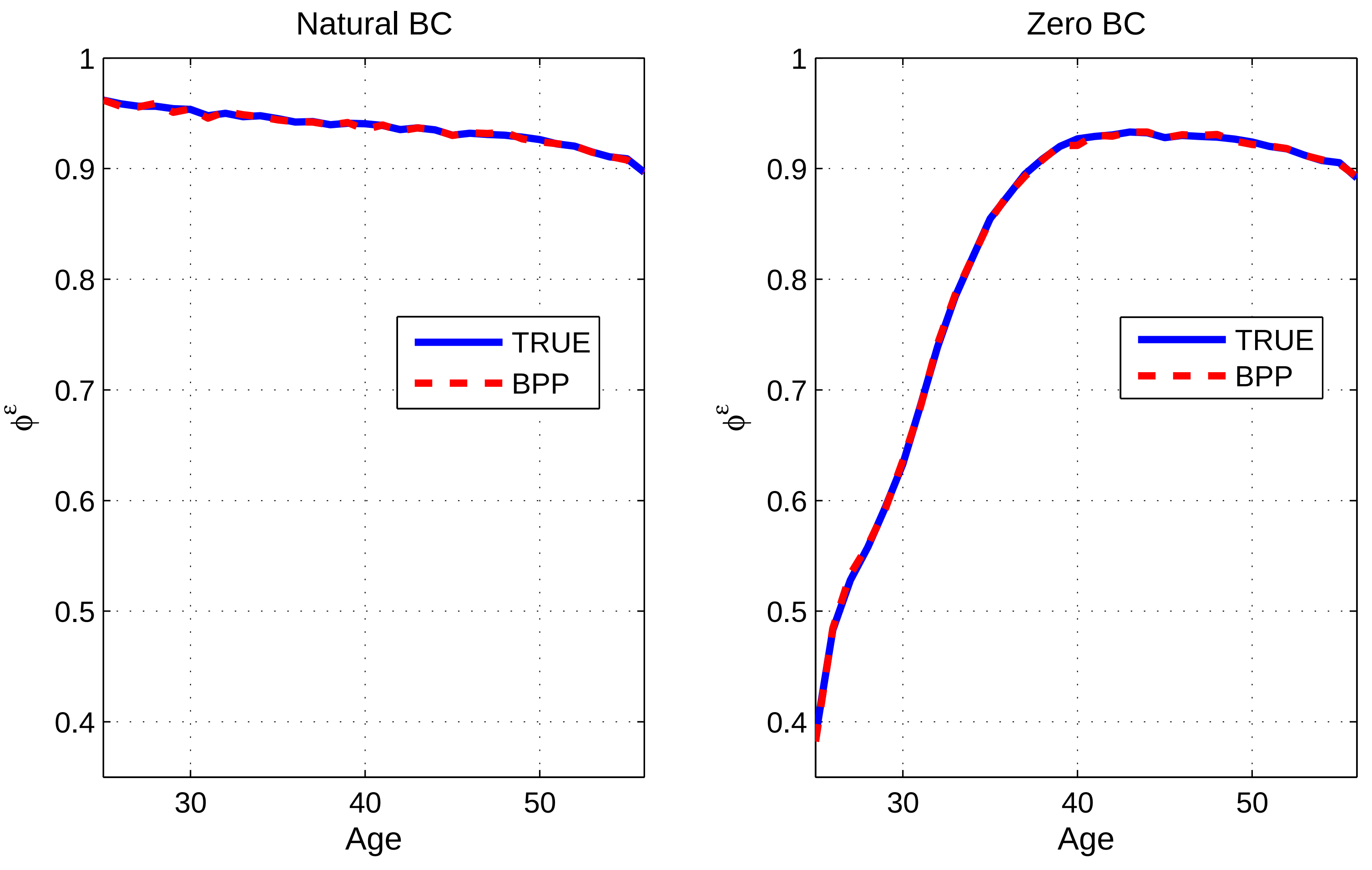
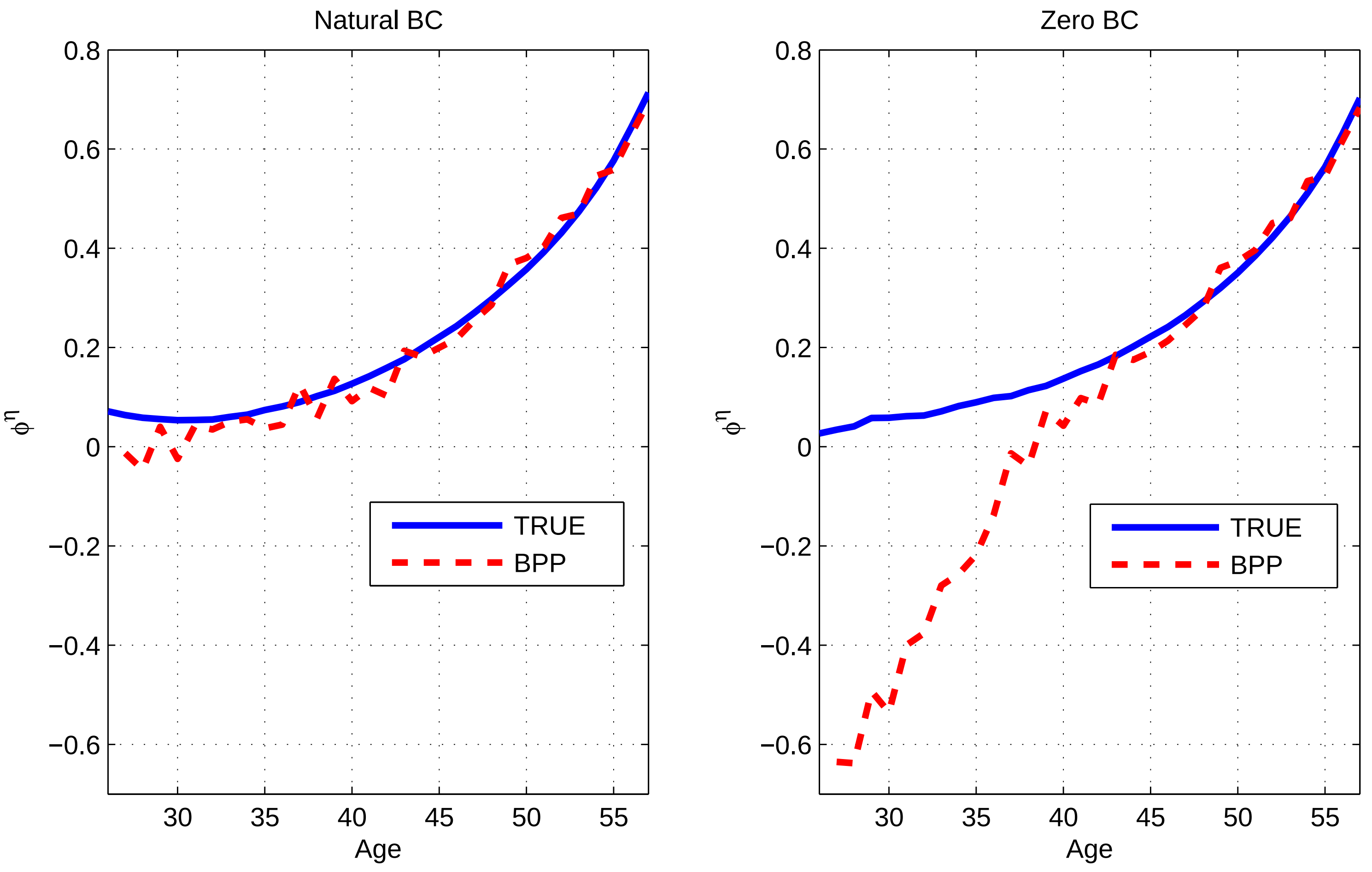
- BPP coefficient for transitory shocks are unbiased
- BPP coefficient for permanent shocks are downward biased
  - Bias massive for no-borrowing economy

Age profile of ϕ^{ε}



- Ability to borrow crucial to smooth transitory shocks at young ages

Age profile of ϕ^η



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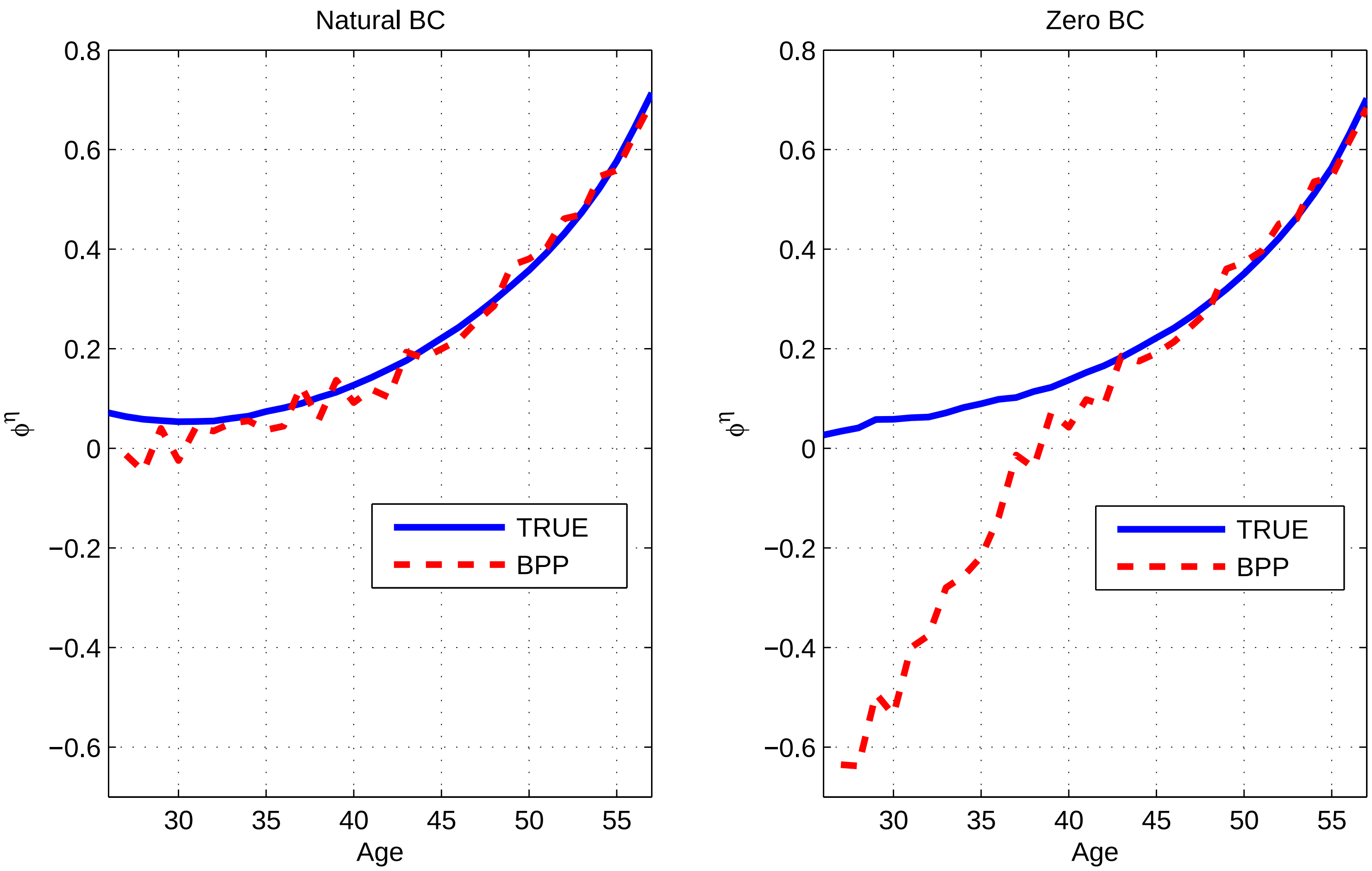
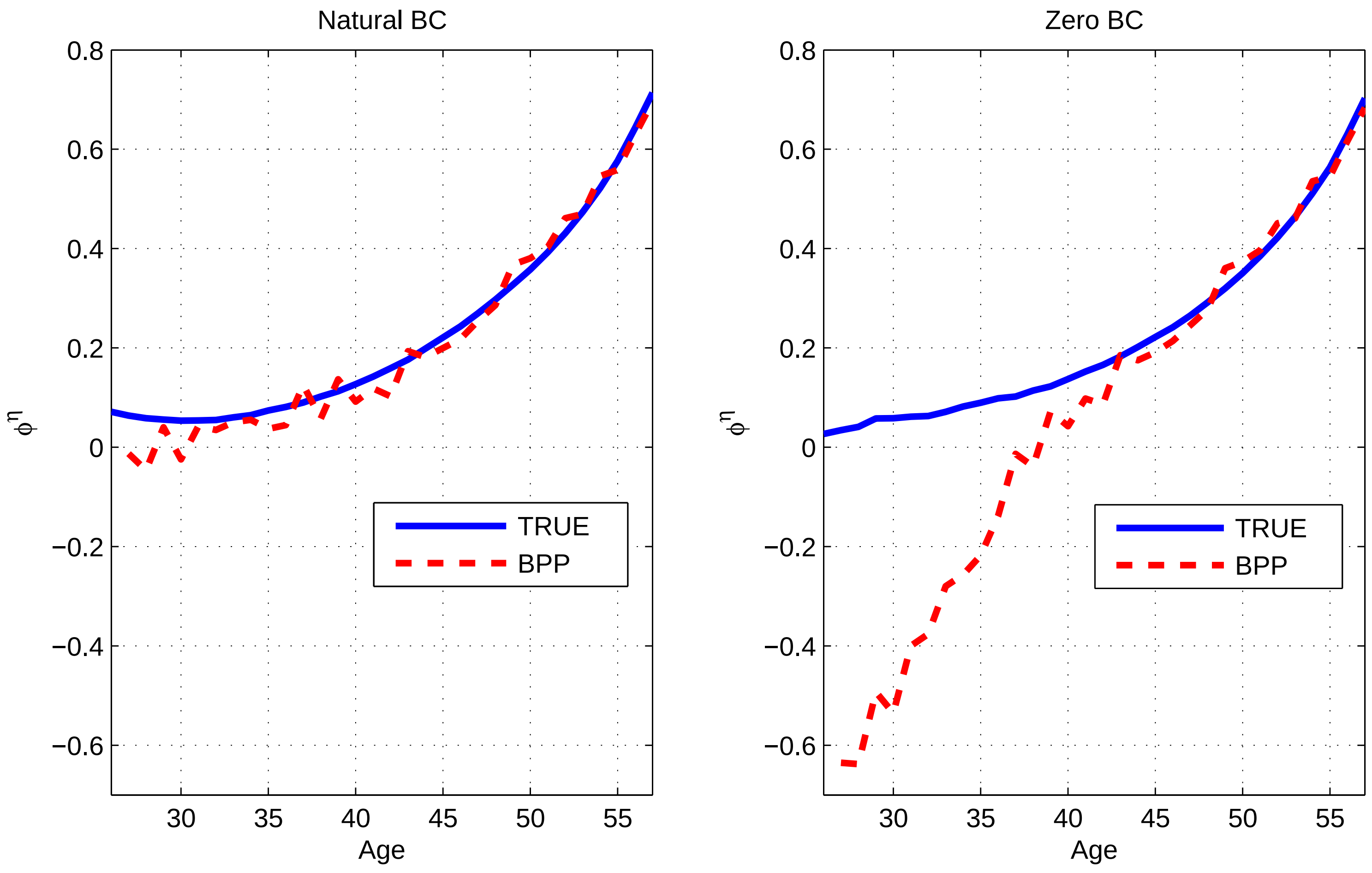


- Age profile of insurance coefficients against permanent shocks (ϕ_t^η) in the model is increasing, whereas in the data it is flat

Age profile of ϕ^η



- Bias in BPP estimator large when agents are close to the constraint

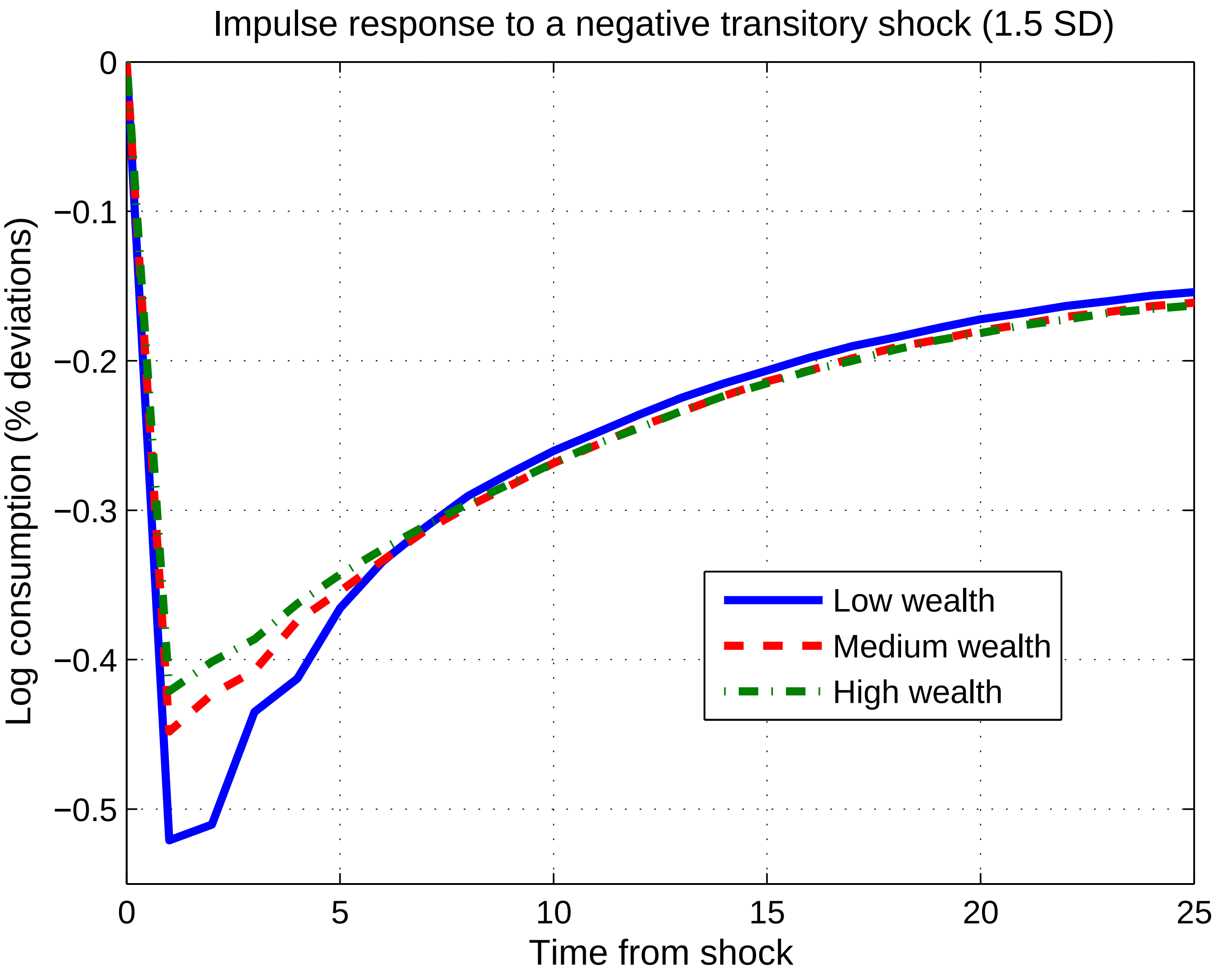
Why the Downward Bias in BPP Estimator?

- From the definition of ϕ^{η}_{BPP}:

$$\begin{aligned}
\phi^{\eta}_{BPP} &= 1 - \frac{cov\left(\Delta c_{it}, \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}\right)}{cov\left(\Delta y_{it}, \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}\right)} \\
&= 1 - \frac{cov\left(\Delta c_{it}, \eta_{i,t-1} + \varepsilon_{i,t-2} + \eta_{it} + \eta_{i,t+1} + \varepsilon_{i,t+1}\right)}{var\left(\eta_{it}\right)} \\
&= \phi^{\eta} + \underbrace{\frac{cov\left(\Delta c_{it}, \eta_{i,t-1} + \varepsilon_{i,t-2}\right)}{var\left(\eta_{it}\right)}}_{\text{A2: short memory}} + \underbrace{\frac{cov\left(\Delta c_{it}, \eta_{i,t+1} + \varepsilon_{i,t+1}\right)}{var\left(\eta_{it}\right)}}_{\text{A1: no adv. info}} \\
&= \phi^{\eta} + \underbrace{\frac{cov\left(\Delta c_{it}, \varepsilon_{i,t-2}\right)}{var\left(\eta_{it}\right)}}_{\ll 0}
\end{aligned}$$

- Last term large when agent close to borr. constr. at $t-2$

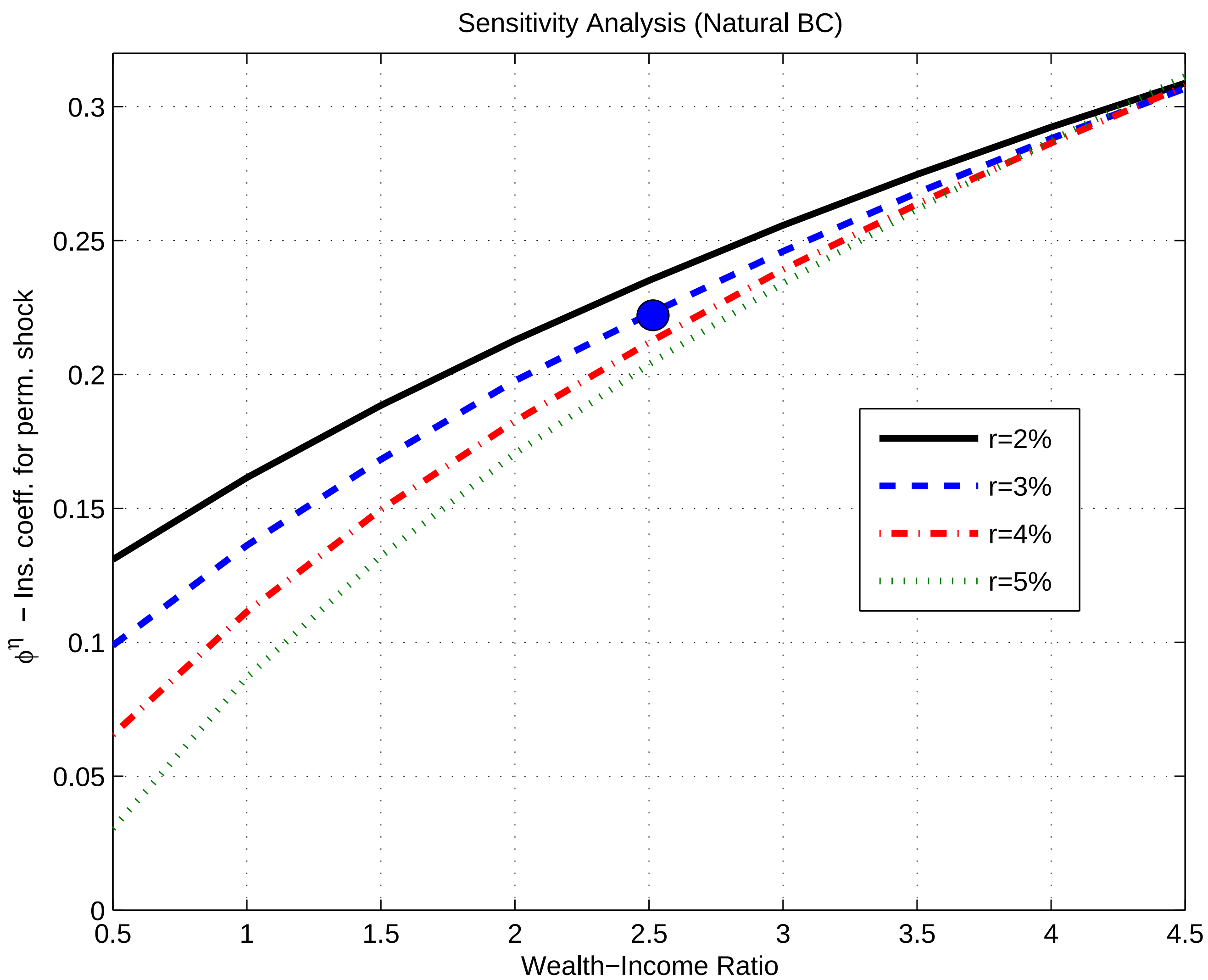
An "Impulse Response"



Sensitivity Analysis (Natural BC)

	Permanent Shock		Transitory Shock	
	TRUE (0.24)	BPP (0.22)	TRUE (0.94)	BPP (0.94)
Initial Wealth Dist.	0.24	0.23	0.94	0.94
$\gamma = 5$	0.27	0.25	0.93	0.93
$\gamma = 10$	0.32	0.29	0.92	0.92
Rep. ratio $= 0.25$	0.19	0.17	0.93	0.93
Rep. ratio $= 0.65$	0.27	0.26	0.94	0.94
$\sigma_\eta = 0.02$	0.25	0.23	0.93	0.93
$\sigma_\eta = 0.005$	0.23	0.21	0.94	0.94
$\sigma_{z_0} = 0.2$	0.24	0.23	0.94	0.94
$\sigma_{z_0} = 0.1$	0.23	0.22	0.94	0.94
$\sigma_\varepsilon = 0.075$	0.24	0.22	0.94	0.94
$\sigma_\varepsilon = 0.025$	0.23	0.22	0.94	0.94

Sensitivity Analysis (K/Y and r)



Advance Information

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- Model I: households observe, one period in advance, a fraction of the permanent shock
- Model II: households know their own deterministic income profile at age $t = 0$ (e.g., Lillard-Weiss, 1979)
- Given BPP identification method, neither form of advance information can reconcile model and data

Preempting the permanent shock

- Permanent income growth in period t comprises of two orthogonal additive components, η_{it}^{s} and η_{it}^{a}
- The component η_{it}^{a} is already in the information set of the agent at time $t-1$

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- Permanent income growth in period t comprises of two orthogonal additive components, η_{it}^{s} and η_{it}^{a}
- The component η_{it}^{a} is already in the information set of the agent at time $t-1$
- From the definition of insurance coefficient:

$$
\begin{aligned}
\phi^{\eta} &= 1 - \frac{cov\left(\Delta c_{it}, \eta_{it}\right)}{var\left(\eta_{it}\right)} = 1 - \frac{cov\left(\Delta c_{it}, \eta_{it}^{s} + \eta_{it}^{a}\right)}{var\left(\eta_{it}^{s} + \eta_{it}^{a}\right)} \\
&= \frac{var\left(\eta_{it}^{s}\right)}{var\left(\eta_{it}\right)} \phi^{\eta^{s}} + \frac{var\left(\eta_{it}^{a}\right)}{var\left(\eta_{it}\right)} \left[1 - \frac{cov\left(\Delta c_{it}, \eta_{it}^{a}\right)}{var\left(\eta_{it}^{a}\right)}\right] \\
&\approx \left(1-\alpha\right) \phi^{\eta^{s}} + \alpha
\end{aligned}
$$

increasing in α, since with loose borrowing limits $cov\left(\Delta c_{it}, \eta_{it}^{a}\right) \approx 0$

Preempting the permanent shock

- Ignoring the usual downward bias, the BPP methodology yields:

$$
\begin{aligned}
\phi^{\eta}_{BPP} &= 1 - \frac{cov\left(\Delta c_{it}, \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}\right)}{cov\left(\Delta y_{it}, \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}\right)} \\
&= 1 - \frac{cov\left(\Delta c_{it}, \eta^{s}_{it} + \eta^{a}_{i,t} + \eta^{a}_{i,t+1}\right)}{var\left(\eta^{s}_{it} + \eta^{a}_{it}\right)} \\
&\approx (1-\alpha)\,\phi^{\eta^{s}} + \alpha\left[1 - \frac{cov\left(\Delta c_{it}, \eta^{a}_{i,t+1}\right)}{var\left(\eta^{a}_{it}\right)}\right] \\
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\end{aligned}$$

- BPP estimator is independent of the amount of advance information

- Simulations confirm this finding

Predictable individual income profile

Predictable individual income profile

- Generalize log-earnings (deviations from common age-profile) to:

$$\begin{aligned} y_{it} &= \beta_i t + z_{it} + \varepsilon_{it} \\ z_{it} &= z_{i,t-1} + \eta_{it}, \end{aligned}$$

  with $E[\beta_i] = 0$ in the cross-section, and $SD[\beta_i] = \sigma_\beta$

- The individual-specific slope β_i is learned at time zero

- Lillard-Weiss (1979), Baker (1997), Haider (2001), Guvenen (2007)

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- When we increase σ_β, we decrease σ_η accordingly to keep the total rise in lifetime earnings inequality constant

Predictable individual income profile

	Permanent Shock		Transitory Shock	
Data	0.36 (0.09)		0.95 (0.04)	
	Model TRUE	Model BPP	Model TRUE	Model BPP
Natural BC				
40%	0.24	0.25	0.94	0.94
60%	0.24	0.28	0.94	0.94
80%	0.24	0.37	0.94	0.94
Zero BC				
40%	0.24	-0.01	0.82	0.82
60%	0.24	-0.10	0.82	0.82
80%	0.24	-0.31	0.82	0.82

- Upward bias in BPP coefficient with natural BC

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- Additional downward bias in BPP coefficient with zero BC

Why the Upward Bias in the BPP Estimator?

- From the definition of ϕ^{η}_{BPP}:

$$\phi^{\eta}_{BPP} = 1 - \frac{cov\left(\Delta c_{it}, \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}\right)}{cov\left(\Delta y_{it}, \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}\right)}$$

$$= 1 - \frac{cov\left(\Delta c_{it}, \eta_{i,t-1} + \varepsilon_{i,t-2} + \eta_{it} + \eta_{i,t+1} + \varepsilon_{i,t+1} + 3\beta_i\right)}{var\left(\eta_{it}\right) + 3var\left(\beta_i\right)}$$

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- Ignoring usual downward bias due to binding constraint:

$$\phi^{\eta}_{BPP} \approx \left[\frac{var\left(\eta_{it}\right)}{var\left(\eta_{it}\right) + 3var\left(\beta_i\right)}\right]\phi^{\eta} + \left[\frac{3var\left(\beta_i\right)}{var\left(\eta_{it}\right) + 3var\left(\beta_i\right)}\right]\left[1 - \frac{cov\left(\Delta c_{it}, \beta_i\right)}{var\left(\beta_i\right)}\right]$$

$$= \left(1 - \alpha\right)\phi^{\eta} + \alpha\phi^{\beta}$$

$\phi^{\beta} \approx 1$ with loose borrowing constraints (upward bias)

$\phi^{\beta} \approx 0$ with tight borrowing constraints (downward bias)

Persistent (rather than permanent...) shocks

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- Generalize log-earnings process to AR(1) + transitory:

$$\begin{aligned} y_{it} &= z_{it} + \varepsilon_{it} \\ z_{it} &= \rho z_{it-1} + \eta_{it}, \quad \text{with } \rho < 1 \end{aligned}$$

- BPP instruments no longer valid [misspecification]

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- BPP instruments no longer valid [misspecification]

- Define quasi-difference: $\tilde{\Delta} y_t \equiv y_t - \rho y_{t-1}$

- Identification of $(\phi^{\eta}, \phi^{\varepsilon})$ can still be achieved by setting

$$\begin{aligned} g_t^{\varepsilon}(\mathbf{y}_i) &= \tilde{\Delta} y_{t+1} \\ g_t^{\eta}(\mathbf{y}_i) &= \rho^2 \tilde{\Delta} y_{t-1} + \rho \tilde{\Delta} y_t + \tilde{\Delta} y_{t+1} \end{aligned}$$

under same assumptions A1 & A2

Persistent shocks

	Persistent Shock			Transitory Shock		
Data	$0.36\ (0.09)$			$0.95\ (0.04)$		
	TRUE	BPP	BPP (missp.)	TRUE	BPP	BPP (missp.)
Natural BC						
$\rho = 0.99$	0.31	0.29	0.28	0.93	0.93	0.93
$\rho = 0.97$	0.41	0.39	0.39	0.93	0.92	0.92
$\rho = 0.95$	0.48	0.46	0.46	0.92	0.92	0.90
Zero BC						
$\rho = 0.99$	0.28	0.18	0.17	0.82	0.82	0.82
$\rho = 0.97$	0.34	0.27	0.27	0.82	0.82	0.80
$\rho = 0.95$	0.38	0.35	0.33	0.81	0.81	0.79
$\rho = 0.93$	0.42	0.40	0.38	0.81	0.81	0.78

- Reconciliation of model and data for $\rho \in (0.93, 0.97)$

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- Misspecification bias in BPP estimator is small

Persistent shocks

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- Usual downward bias in BPP estimator

Persistent shocks

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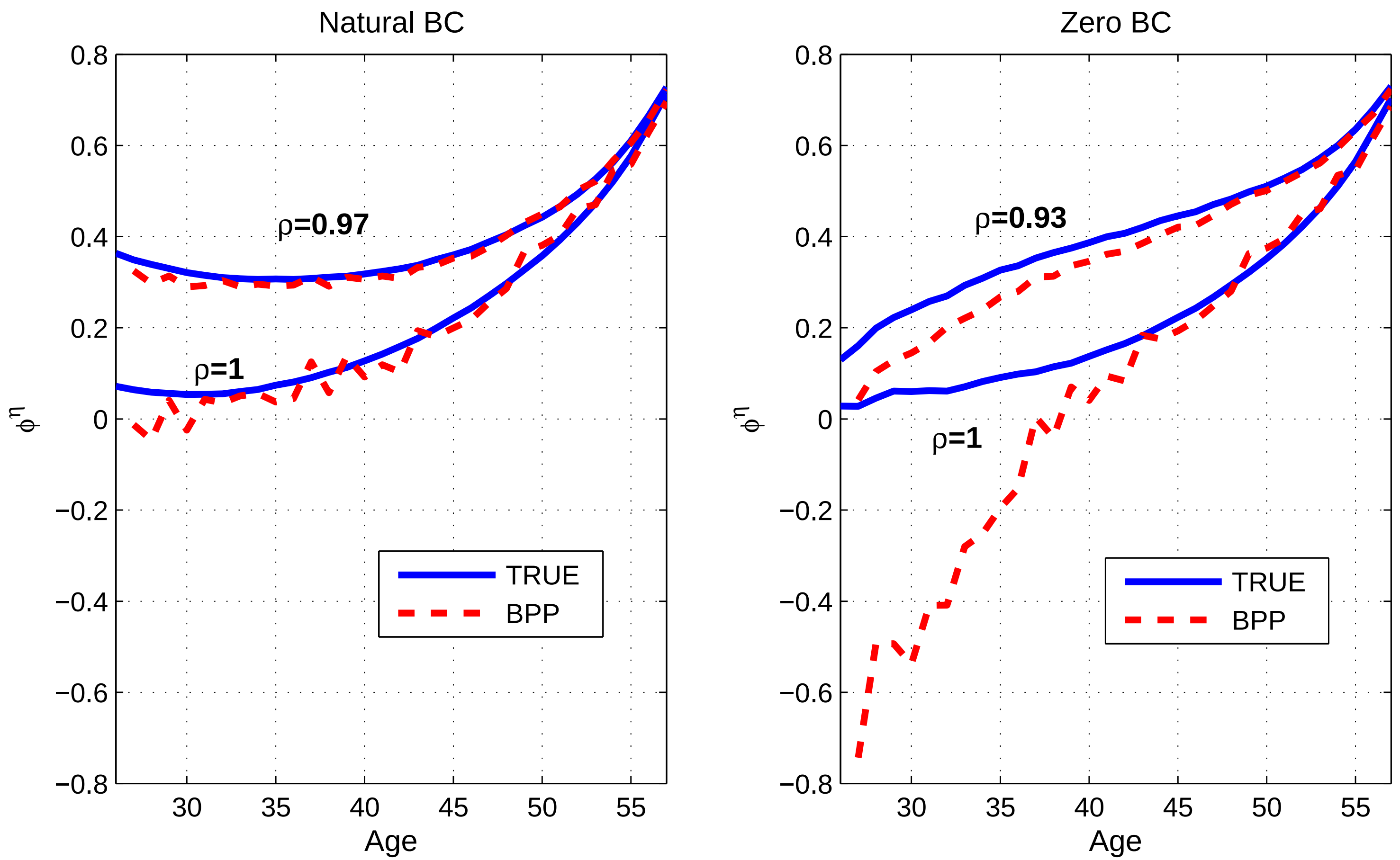
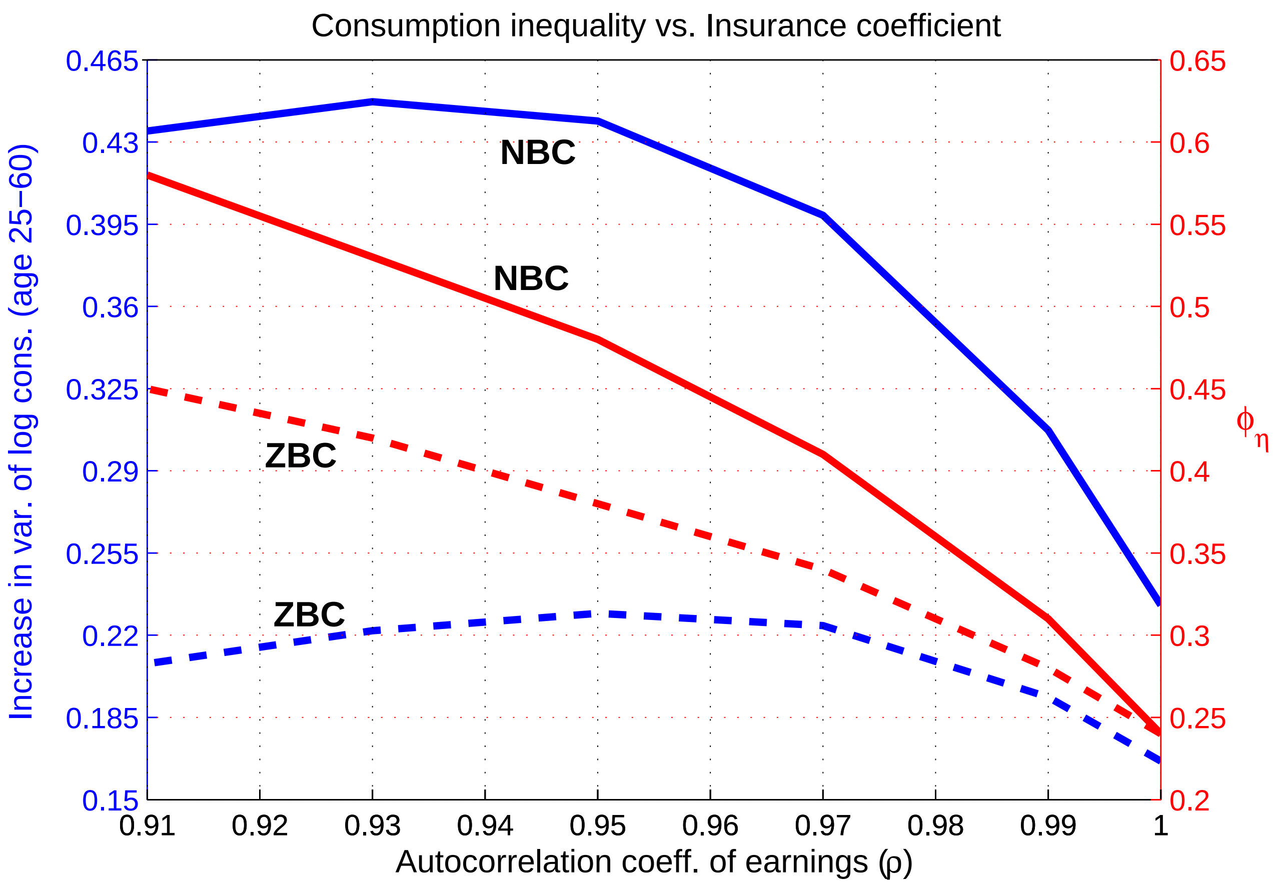
- Insurance coefficients for transitory shocks unaffected

Age profile of ϕ^η

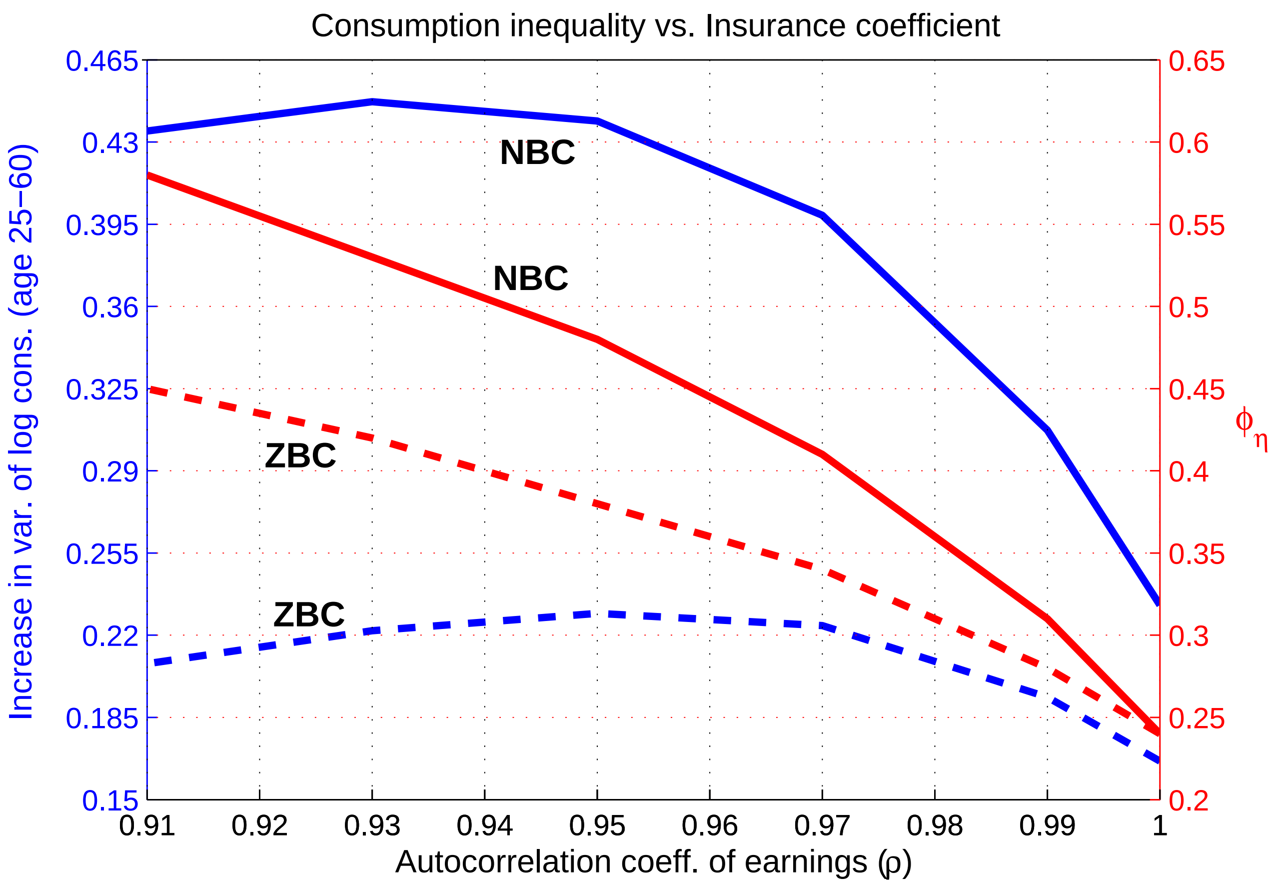


- In the model, age profile of insurance coefficients wrt to persistent shocks is flatter, hence closer to the data

Relationship with STY

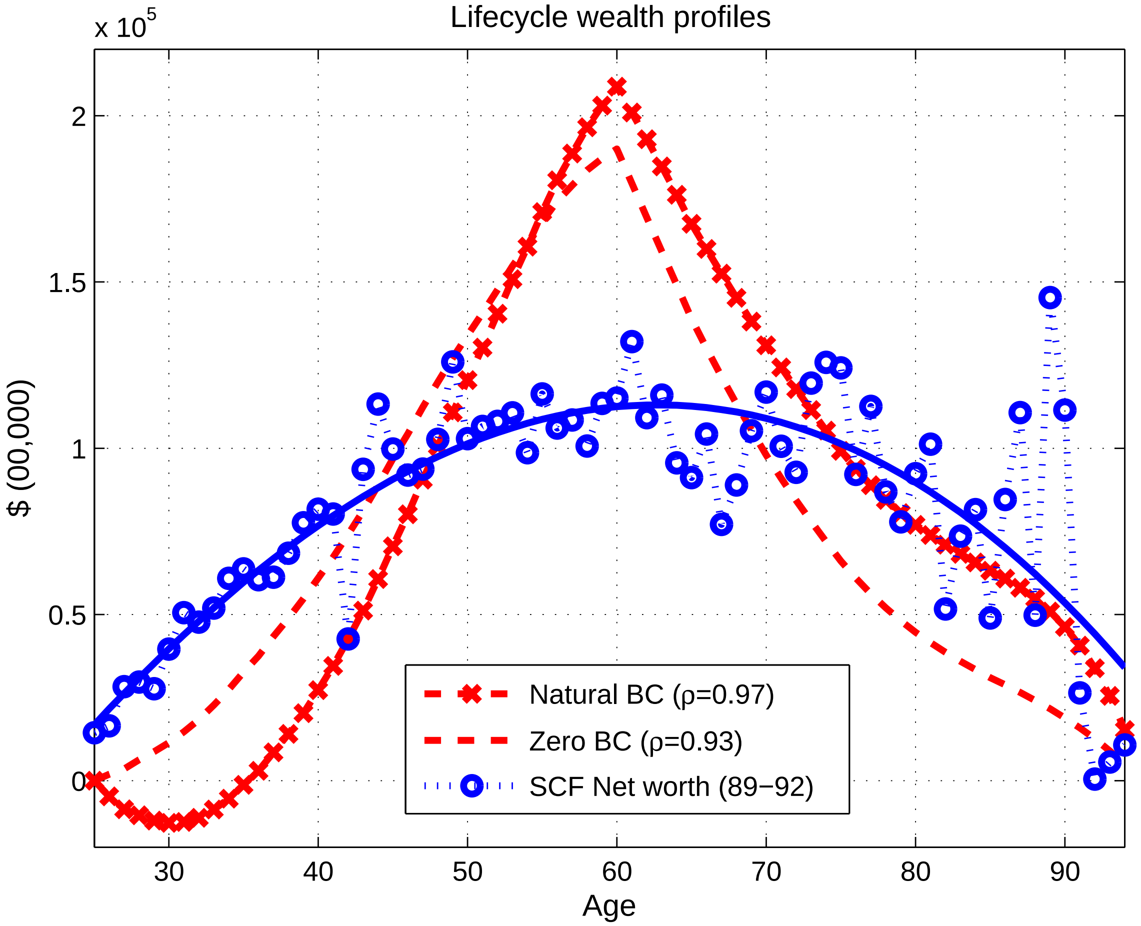


Relationship with STY

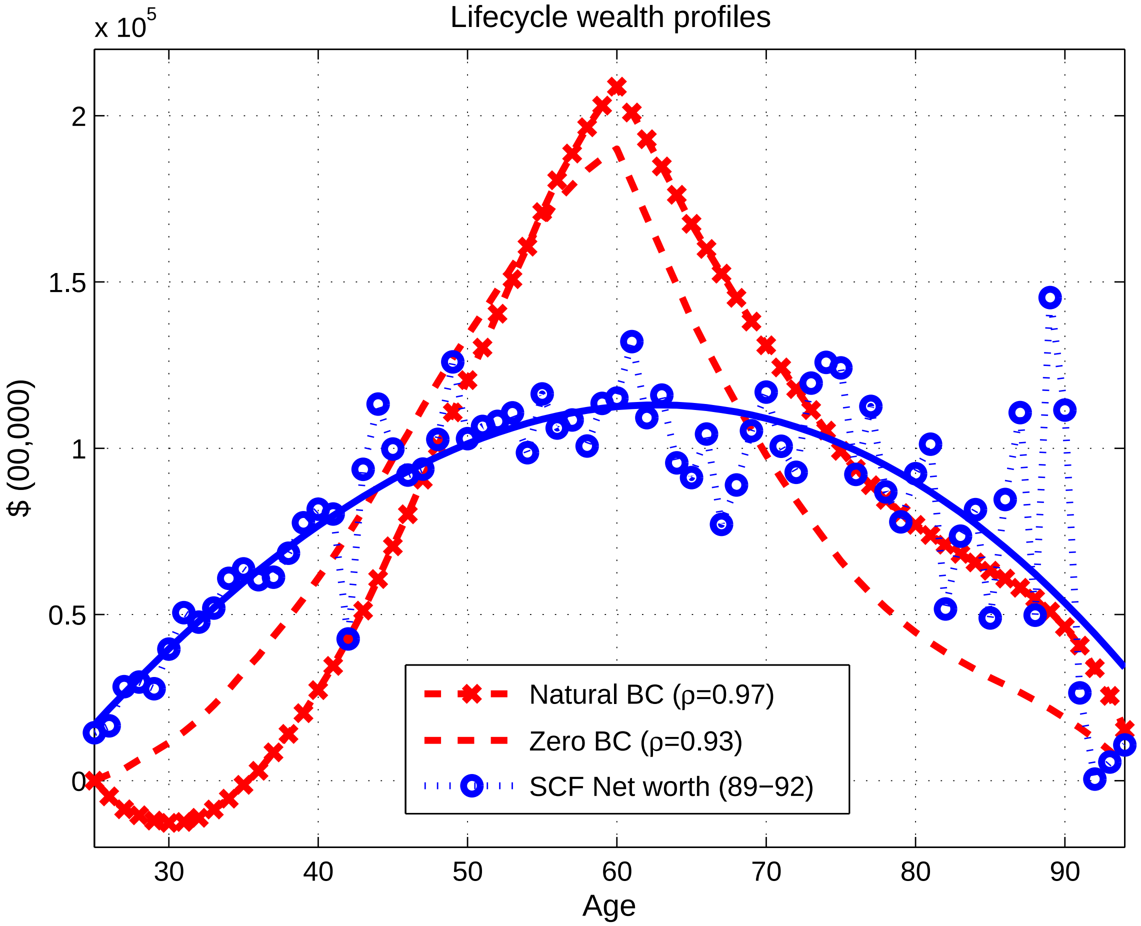


- The size of the rise in consumption inequality over the lifecycle is an imperfect proxy for consumption insurance

Age profile of wealth: model vs. data



Age profile of wealth: model vs. data



- A version of the model with more realistic age profile of wealth would be also more successful in replicating the BPP facts

Conclusions

1. We generalized BPP methodology, and argued that insurance coefficients should become a key summary statistic of IM models

2. BPP estimator downward biased when BC tight

3. Plausibly calibrated Bewley model has too little insurance

4. Ins. coeff. $\neq$ rise in consumption inequality over life cycle

5. Advance information does not reconcile model and data

6. A (very) persistent income shock goes a long way

7. Modifications of model that get age-wealth profile right promising