

The Science and Art of DSGE Modelling

A Foundations Course

Global Sensitivity Analysis

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Global Sensitivity Analysis Toolbox

- Global Sensitivity Analysis toolbox.

Readings:

- Ratto, M.. (2008). "Analysing DSGE Models with Global Sensitivity Analysis", *Computational Economics* 31 (2):115-139.
- Ratto, M., Hohberger, S., Burgert, M. (2016). "New Software Tool for the Evaluation of DSGE Models: An Application with DYNARE", mimeo.

Global Sensitivity Analysis and Monte-Carlo-Filtering. Ratto(2008)

- Which is the domain of structural coefficients assuring the stability and determinacy of a DSGE model?
- Which parameters mostly drive the fit of, e.g. GDP and which the fit of inflation?
- How to represent in a direct, albeit approximated, form the relationship between structural parameters and the reduced form of a rational expectations model?
- We can use computational tools called GSA (and it's Dynare toolbox) to answer these questions.

Global Sensitivity Analysis and Monte-Carlo-Filtering. Ratto (2008)

- Think about the stability/indeterminacy analysis.
- Usually papers produce a 2 dimensional analysis varying 2 parameters at the time.
- Here we will show how it is possible to work out the determinacy/stability region of a model more generally than 2 parameters at the time.
- [?] discusses two approaches: Monte Carlo Filtering (MCF) techniques and Functional variance decomposition techniques.
- We will focus here only on the first.

Monte-Carlo-Filtering. Ratto (2008)

- Is special type of mapping procedure that is applied when the objective of the analysis is to measure what fraction of a set of Monte Carlo model realisations (a forecast, an IRF, a stochastic simulation) falls within established bounds or regions.
- A sensitivity analysis procedure called regionalised sensitivity analysis (RSA) is sub-sequently applied in order to know which input factors are mostly responsible for the model to fall under the prescribed region or not.
- MCF is often used in calibration, as it can successfully point out to the existence of alternative behavioural regions in the multidimensional space of the structural parameters.

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

- These techniques can be used to map the stability region of DSGE models and to detect critical regions in the model parameter space of DSGE models.
- This is usually done by solving the model for parameter values on some grid. Like we did before.
- MCF generalizes those, in a Bayesian context, and allows us to deal with entire multivariate parameter spaces.
- A multi-parameter MC simulation is performed, sampling model parameters (X_1, \dots, X_k) from prior ranges and propagating parameter values through the model.

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

- Then, based on a set of constraints (e.g. rank conditions or signs of impulse responses), a categorization is defined for each MC model realization as lying either within or outside the target region.
- The terms behavior (B) or non-behavior (\bar{B}) are used in the MCF literature. The $B - \bar{B}$ categorization is mapped back onto the input structural parameters, each of which is thus also partitioned into a B and \bar{B} sub-sample.
- Given a full set of N Monte Carlo runs, one obtains two subsets: $(\theta_i|B)$ of size n and $(\theta_i|\bar{B})$ of size \bar{n} , where $n + \bar{n} = N$.
- In general, the two sub-samples will come from different unknown probability density functions: $f_n(\theta_i|B)$ of size n and $f_{\bar{n}}(\theta_i|\bar{B})$ of size \bar{n} .
- In order to identify the parameters that mostly drive the DSGE model into the target behavior, the distributions f_n and $f_{\bar{n}}$ are compared for each parameter independently.

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

- The Montecarlo sampling allows us to avoid computing analytical integration over the remaining parameters.
- If for a given parameter X_i the two distributions are significantly different, then X_i is a key factor driving the model behavior and there will be clearly identifiable subsets of values in its predefined range that are more likely to fall under B than under \bar{B} .
- If the two distributions are not significantly different, then X_i is unimportant and any value in its predefined range is likely to fall either in B than under \bar{B} .
- Ideally, we are comparing the supports of the conditional cumulative distribution functions (CDF) of a parameter and compute the distance under standard statistical metrics.

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

- The Smirnov two-sample test (two-sided version) provides us with a statistical concept of distance.

$$d_{n,\bar{n}}(X_i) = \sup ||F_n(X_i|B) - F_{\bar{n}}(X_i|\bar{B})||$$

- And the question answered by the test is: “At what significance level α does the computed value of $d_{n,\bar{n}}$ determines the rejection of the null hypothesis $F_n(X_i|B) = F_{\bar{n}}(X_i|\bar{B})$?”
- The lower the α associated to the Smirnov test, the more likely is to reject the null hypothesis.
- The B and \bar{B} subsets can be further inspected through bi-dimensional projections, in order to detect patterns characterizing two-way interactions.
- The standard procedure consists of computing the correlation coefficients ρ_{ij} between all parameters under the B and \bar{B} subsets, and plotting the bi-dimensional projections of the sample for the couples having $|\rho_{ij}|$ larger than a significance threshold.

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

- The literature has mainly focused on sensitivity exercises on calibrated parameters where the model objects of interest are computed by varying one parameter at a time.
- The MCF has clear advantages over calibration sensitivity exercises.
- First, unlike sensitivity calibration exercises, all parameters move simultaneously.
- Second, the Smirnov test offers implicitly a statistical ranking of parameters from the most to the least influential.
- Finally, it unveils important relationships among parameters. We will use these insights to assess the prior model ability to replicate the key data facts and try to uncover the deep parameters that are important to match such patterns.

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

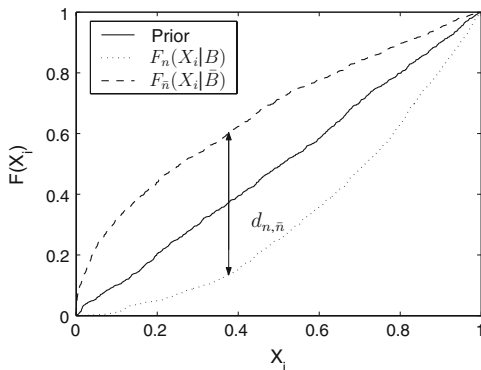


Fig. 1 Graphical representation of the Smirnov test for an important parameter X_i , sampled from a uniform distribution in the interval $[0, 1]$. The vertical bar is the $d_{n,\bar{n}}$ statistic. Model realisations under the B category are more likely when X_i falls on the right part of its pre-defined range

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

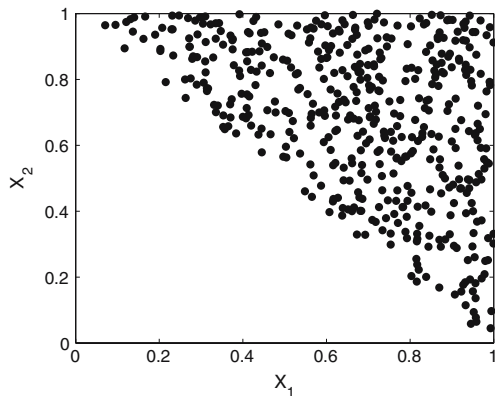


Fig. 2 Bi-dimensional projection of the B sub-sample for the simple model $Y = X_1 + X_2$ (target behaviour $Y > 1$). The triangular pattern clearly indicates the relationship $X_1 + X_2 > 1$ for the target behaviour. The correlation coefficient between X_1 and X_2 in the MC sample is $\rho_{12} = -0.45$

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

- Now let's use the GSA toolbox to reproduce the same stability analysis of the NK model we looked at earlier today.
- This morning we could only do a 2-dimensional analysis. I.e. producing a plot of the interest rate smoothing vs the inflation response in the Taylor rule.
- Now we can check the Stability region for multiple parameters at the same time.

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

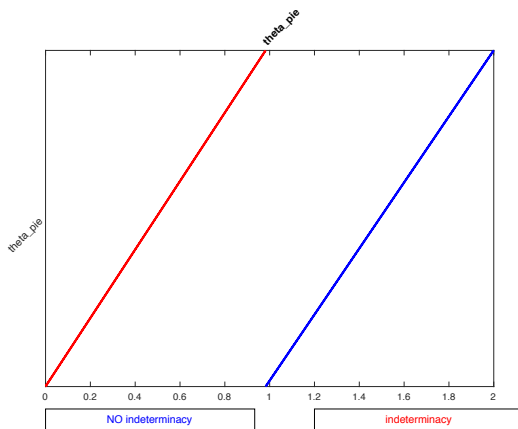


Figure: NK linear code. Determinacy region of θ_π vs ρ^r

Monte-Carlo-Filtering and Determinacy/Stability. Ratto (2008)

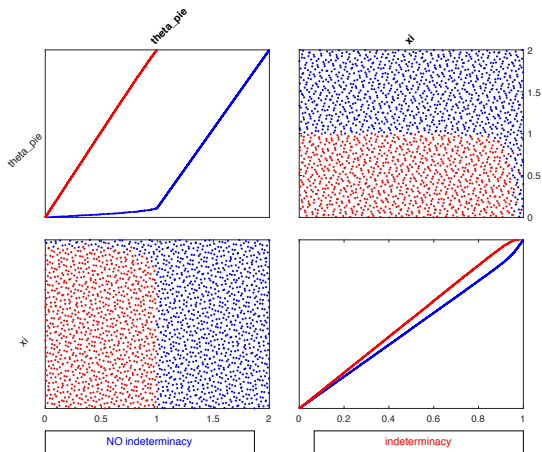


Figure: NK linear code. Determinacy region of θ_{π} vs ρ^r vs ξ

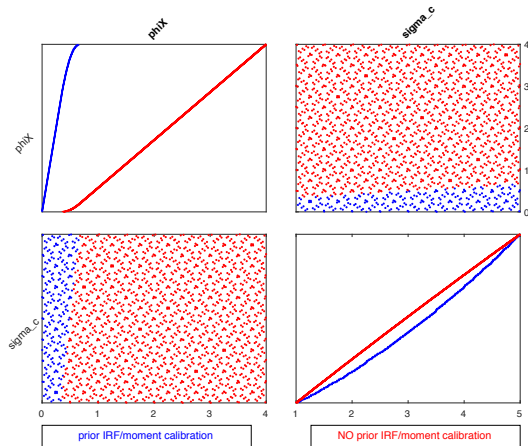
Monte-Carlo-Filtering options

- Code **NKlinear.mod** in folder **GSA** reproduces the figures above.
- There are various advanced options available for the command *dynare_sensitivity*
- Nsam=2048; size of the MC sample.
- pvalue_ks=0.001; critical p-value for the smirnoff statistic d; plot parameters with p-value<pvalue_ks.
- For more options see the Dynare manual.

Monte-Carlo-Filtering applied to IRFs

- MCF can be applied to other exercises.
- For example B and \bar{B} can represent other things instead of determinacy and indeterminacy regions.
- Let's apply it to check the parameters responsible to the switch in sign of a particular Impulse response to a structural shock in DSGE models.
- We will use as an example the Hours-Technology debate. Which parameter drives the response of hours to technology shocks in the RBC model?
- Run code **RBC.mod** in folder **GSA**

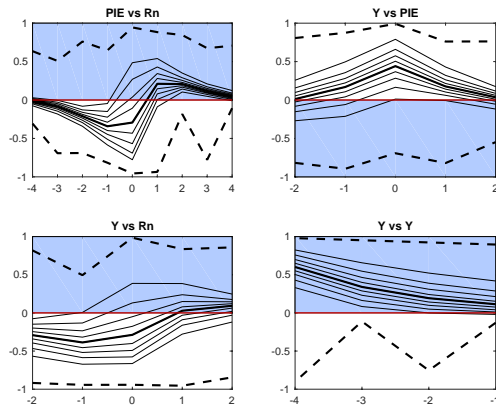
RBC threshold for Hours in response to technology shocks



Monte-Carlo-Filtering applied to Moment Matching

- The second new sensitivity tool within the GSA toolbox is moment calibration that allows for matching or restricting theoretical moments (auto correlations, cross correlations, sign restrictions).
- The idea behind moment restriction is the question whether relationships between two endogenous variables (y_t and x_t) in a DSGE model can be matched with those in the data.
- In code **NK_Linear_moments.mod** in folder **GSA** we are interested in the auto correlation function (ACF) of output Y as well as three cross correlation functions (CCF) between two variables, namely (1) inflation and interest rate, (2) output and inflation and (3) output and interest rate.
- Additionally, we restrict the theoretical moments either to be positive (1 and 3) or negative.
- Note that we specify priors distributions as the ones you will be using tomorrow for estimation.

Monte-Carlo-Filtering applied to Moment Matching



Monte-Carlo-Filtering applied to Moment Matching

- The targeted regions/restrictions are marked in blue.
- Let's focus on the first subplot which shows the correlation between inflation (PIE) and nominal interest rate (R_n).
- It plots the range of cross correlations starting from $R(-4)$ on PIE until $R(+4)$.
- Given the MC sample based on the prior distributions the model provides a negative contemporaneous correlation between PIE and R_n . Only correlations between $R(+1)$ and $R(+4)$ with PIE are inside the targeted region.