FINANCIAL FRICTIONS IN DSGE MODELS

Bayesian Estimation of the NK, GK and BGG Models

Afrasiab Mirza

September 11 2020

From Calibration to Systems Estimation

- Up to now we have used **calibration** to pin down parameters in the model.
- We have used the deterministic steady state to solve for parameter values that result in observed long-run outcomes for macro-variables such as hours worked, the great ratios (consumption, investment shares given a government spending share) and the real interest rate.
- This **first-moment matching** can be extending to a second-moment matching of variances, correlations and autocorrelations to calibrate shock processes.
- Current practice in empirical macroeconomics is to replace this informal moment matching with formal **systems estimation**.
- Maximum Likelihood (ML), General Method of Moments (GMM) and Bayesian estimation are three widely used systems estimation methods for DSGE models.

Overview of Bayesian Estimation

Bayesian analysis requires:

- Initial information $\Rightarrow$ Prior distribution
- Data $\Rightarrow$ Likelihood density or the probability of observing the data given the model and parameters
- Prior and Likelihood $\Rightarrow$ Bayes theorem $\Rightarrow$ Posterior distribution
- Posterior distribution used for confidence intervals for parameters and impulse responses.
- The posterior distribution also provides information regarding identification of parameters - how much information does the data provide on parameters?

Dynare Steps in the Computation

1. Solves the model for a particular parameter vector θ - we'll only consider a first-order (linear) solution
2. Evaluates the likelihood density $p(y|\theta)$ using the linear Kalman filter and assuming Gaussian shocks
3. Maximizes $p(y|\theta)p(\theta)$ numerically to arrive at the mode of θ (repeating 1 and 2 each time)
4. Computes an estimate of covariance matrix of the parameters, $\hat{\Sigma}_\theta$ using the result

$$\hat{\Sigma}_\theta = \left(-\frac{\partial^2 \log(p(y|\theta))}{\partial\theta\partial\theta'} \right)^{-1} \tag{1}$$

   evaluated at the mode. The term in the brackets is the Hessian.
5. Output is reported at this stage (the prior mean, the estimated mode, its standard deviation and a t-test). The user can stop here.
6. Proceeds to the computation of the posterior distribution using MCMC

Summary of Dynare Bayesian Estimation Procedures

1. transform the actual data to fit properties of the model (not in Dynare)
2. specify prior distributions
3. Dynare computes the log-likelihood numerically via the Kalman filter
4. finds the maximum of the likelihood and posterior mode
5. draws posterior sequences and simulates posterior distribution with Metropolis algorithm
6. computes various statistics on the basis of the posterior distribution (post. moments)
7. estimates the *posterior marginal density* (see next slides) to compare models
8. One can examine sensitivity of the results to choice of priors.
9. But before estimation the we need to examine the possibility that some parameters may not be **identified.**

Identification

- The default syntax for the identification procedures in DYNARE:

```
varobs dy pinfobs robs;
identification(advanced=1);
```

- point identification at the prior mean
- the MC exploration using the draws from the prior distribution
- identification strength measured at the mean and weighted by the prior standard deviation.

Identification Example: NK Model

- The output of this procedure then reveals whether or not there are identification problems with parameter estimation:

```
==== Identification analysis ====
Testing prior mean
Testing prior mean
REDUCED-FORM: All parameters are identified in the Jacobian of
steady state and reduced-form solution matrices (rank(Tau) is f
with tol = robust).
MINIMAL SYSTEM (KOMUNJER AND NG, 2011): All parameters are iden
in the Jacobian of steady state and minimal system (rank(Deltab
is full with tol = robust).
```

- So far so good.

Identification Example: NK Model

- Continuing with the identification tests:

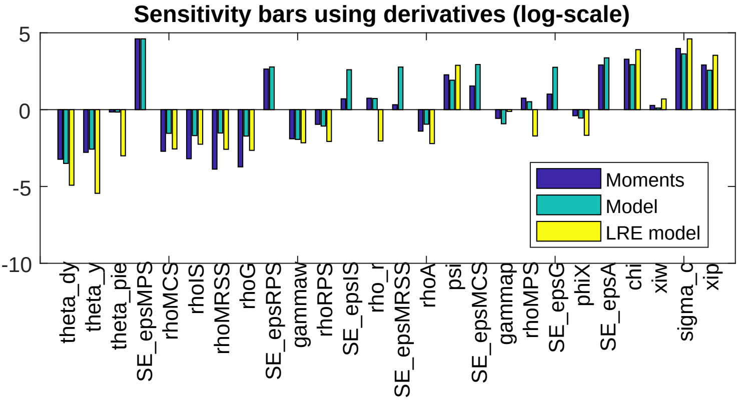
```
SPECTRUM (QU AND TKACHENKO, 2012): !!!WARNING!!! The rank of Gb
(Jacobian of mean and spectrum) is deficient by 6 (rank(Gbar) =
20 < 26 with tol = robust)!
theta_dy is not identified! rhoA is not identified! rhoG is not
identified! rhoMCS is not identified! rhoMRSS is not identified
rhoIS is not identified!
MOMENTS (ISKREV, 2010): All parameters are identified in the Ja
of first two moments (rank(J) is full with tol = robust).
```

- If the rank condition fails in any of these two procedures, the procedure indicates which parameters are responsible for identification problems.

Identification Example: NK model

- The following Figure shows an aggregate measure of how changes in the elements of the parameter vector θ impact on the model moments. The impact is measured locally using the Jacobian.
- The problem is that the derivatives are not scale invariant so not easily comparable. For this reason here Dynare uses a normalization/standardization procedure described in Iskrev Ratto (2011).
- Dynare plots here three different measure of sensitivity. The bars depict the norm of the columns of three different standardized Jacobian matrices for the respective parameter shown on the x-axis. The respective Jacobian refer to
  1. the moments matrix, indicating how well a parameter can be identified due to the strength of its impact on moments
  2. the solution matrices, indicating how well a parameter could in principle be identified if all state variables were observed
  3. the Linear Rational Expectation (LRE) model, indicating trivial cases of non-identifiability due for example to the fact that some parameters always show up as a product in the model equations.

Identification Example: NK model



Identification Example: NK model

- To completely rule out a flat likelihood at the local point one can also check collinearity between the effects of different parameters on the likelihood.
- If there exists an exact linear dependence between a pair and among all possible combinations their effects on the moments are not distinct and the violation of this condition must indicate a flat likelihood and lack of identification.
- The details of collinearity analysis require the advanced analysis option $((advanced = 1))$ which prints the results of the brute force search for the groups of parameters whose columns in the Jacobian matrix best explain each column of the Jacobian (i.e. best reproducing the behaviour of each single parameter).

Identification Example: NK model

```
Collinearity patterns with 1 parameter(s)


Parameter [ Expl. params ] cosn
chi [ sigma_c ] 1.0000000
sigma_c [ chi ] 1.0000000
psi [ gammap ] 0.9999849
xip [ chi ] 0.9998896
gammap [ psi ] 0.9999849
gammaw [ gammap ] 0.9986115
rho_r [ theta_pie ] 0.9977585
theta_pie [ rho_r ] 0.9977585
theta_y [ rho_r ] 0.9974422
SE_epsMPS [ theta_pie ] 0.9972525
xiw [ gammaw ] 0.9869263
rhoMPS [ theta_pie ] 0.9705889
```

Identification GK and BGG models

- Very different results are obtained for the GK and BGG models
- Most parameters are reported as unidentified and the advance option that produces the identification strength and collinearity results breaks down.
- However the estimated mode is of good quality as chosen in the mode check, the mcmc procedure converges and the posterior estimates are not centred on the priors.
- So our estimated results look satisfactory.

Estimation of NK Model: Measurement Equations

- We use first-differences of non-stationary data and levels elsewhere. (The Dynare command prefilter=1 demeans the data).
- Inflation is in gross terms and is computed as the ratio of a price index P in two subsequent periods: $\Pi_t^{obs} = \frac{P_t}{P_{t-1}}$
- In net terms, inflation can be approximated by the log of gross inflation, $\pi_t^{obs} = \log \Pi_t^{obs}$ or, equivalently, $\pi_t^{obs} = \log P_t - \log P_{t-1}$
- Interest rates (unlike inflation) are measured net and in annual terms, while models are usually in quarterly terms - if the raw data is in annualised percentage points, then you typically divide the data by 400
- If using a filter, the resulting series will have a zero mean, whereas with first-differences you will be left with the series' average growth rate
- If the model is entered in log-linearized form, then the filtered variables correspond to model variables, as deviations from the steady state

Data

- In the notes you can find all the data sources and transformations used here: for US data, we follow closely Smets and Wouters (2007).

- In the 'Data Preparation' folder you can find the raw data in Excel files, an m-file *data_preparation.m* using different filters, reproducing the figure presented

Measurement Equations - All Models

Estimating the NK, GK and BGG models, US data in first-differences where relevant, the corresponding measurement equations are:

$$\begin{bmatrix} D(log(GDP_t)) = dy \\ D(log(CONS_t)) = dc \\ D(log(I_t)) = di \\ D(log(W_t)) = dw \\ D(log(labobs_t)) = labobs \\ D(log(GDPDEF_t) = pinfobs \\ FEDFUNDS_t/4 * 100 = robs \\ Spread = spreadbcc \end{bmatrix} = \begin{bmatrix} \log\left(\frac{Y_t}{\overline{Y}_t}\right) - \log\left(\frac{Y_{t-1}}{\overline{Y}_{t-1}}\right) + \text{trend growth} \\ \log\left(\frac{C_t}{\overline{C}_t}\right) - \log\left(\frac{C_{t-1}}{\overline{C}_{t-1}}\right) + \text{trend growth} \\ \log\left(\frac{I_t}{\overline{I}_t}\right) - \log\left(\frac{I_{t-1}}{\overline{I}_{t-1}}\right) + \text{trend growth} \\ \log\left(\frac{W_t}{W}\right) - log\left(\frac{W_{t-1}}{W}\right) \\ \log\left(\frac{\Pi_t}{\Pi}\right) + \text{constant}_{\Pi} \\ \log\left(\frac{R_{n,t}}{R_n}\right) + \text{constant}_{R_n} \\ R_k - R \end{bmatrix}$$

- Sample: 1966:1-2017:4 which starts at observation 75 in the data file.
- In the estimation that follows spread data is not used which leaves 7 observables and 7 shocks. This is a necessary condition for invertibility of the state space presentation of the RE solution and the validity of the standard perfect information solution (see Fernandez-Villaverde *et al.* (2007) and Levine *et al.* (2019)).

Dynare and Matlab Files

- **NK_financial.mod** estimates the core NK model. This includes identification, Brook-Gelman convergence diagnostics and historical variance decomposition. This .mod file requires a **_steadystate.m** counterpart, with an accompanying **ss_fun.m** solver.
- **GK_financial.mod** (and accompanying steady state files) estimates the GK model with outside equity
- **BGG_financial.mod** (and accompanying steady state files) estimates the BGG model with a normal distribution for the idiosyncratic shock.
- Matlab programs **NK_validation.m** etc compare first and second moments of observed variables with those of the data used for estimation.
- All results are stored in a text file **Results_summary** for each model
- Matlab program **Comparethem.m** in the sub-folder compare computes model odds from log-likelihood values
- Matlab program **acfs_plot.m** plots the sample ACFs and estimated ACFs. Requires subfunctions **acfcomp.m** and **autocov.m**.

Estimation Output

- results from posterior optimization (also for maximum likelihood)
- marginal data density (modified harmonic mean estimator - Geweke, 1999)
- mean and confidence interval from posterior simulation
- graphs with prior, posterior and mode
- graphs of smoothed shocks and smoothed observation errors
- convergence diagnostics (MCMC replications)
- graphs of posterior IRFs (optional)
- all results are stored in **FILENAME_results.mat** (in particular, in structure array *oo_*)
- all MH draws are saved in the subfolder under the path: *FILENAME**metropolis* – post. median

Bayesian Model Comparison

- First consider Bayes Rule

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \propto p(y|\theta)p(\theta)$$

- So far we have not needed the unconditional density $p(y)$ to maximize $p(\theta|y)$ wrt θ.
- This is computed by integrating over the prior distribution to obtain

$$p(y) = \int_{\Theta} p(y|\theta)p(\theta)d\theta$$

- For a particular model i from a number of alternatives, say m_i, we can define a density conditional on this model

$$p(y|m_i) = \int_{\Theta} p(y|\theta, m_i)p(\theta, m_i)d\theta$$

where $p(\theta, m_i)$ is the prior for that model.
- We refer to $p(y|m_i)$ as the **marginal likelihood** associated with model m_i.

Bayes Factor and Model Odds

- Bayesian inference now allows a framework for comparing alternative and potentially misspecified models based on their marginal likelihoods - a "likelihood race"
- Now construct a **Posterior Odds Ratio** (assuming m_i and m_j):

$$PO_{i,j} = \frac{p(m_i|y)}{p(m_j|y)} = \frac{p(y|m_i)p(m_i)}{p(y|m_j)p(m_j)}$$

- Or a **Bayes Factor** (when the prior odds ratio, $\frac{p(m_i)}{p(m_j)}$, is set to unity):

$$BF_{i,j} = \frac{p(y|m_i)}{p(y|m_j)} = \frac{\exp(LL(y|m_i))}{\exp(LL(y|m_j))}$$

defining the log-likelihood

$$LL(y|m_i) \equiv log(p(y|m_i))$$

noting that $x = \exp(\log x)$.

Bayes Factor and Model Odds - cont.

- Given the Bayes factors one can easily compute the model probabilities $p_1, p_2, ...p_n$ for n models. Since $\sum_{i=1}^{n} p_i = 1$:

$$\frac{1}{p_1} = \sum_{i=2}^{n} BF_{i,1}$$

  from which p_1 is obtained. Then $p_i = p_1 BF(i, 1)$ gives the remaining model probabilities

- **modelcomparison.m**, computes these probabilities given the data densities from the competing models

Comparison of NK, GK and BGG Models

- Formal Bayesian comparison of the benchmark NK model with two financial sector models, the GK model with outside equity and the BGG model
- Dynare Files: **NK_financial.mod** (NK), **FK_financial.mod** (GK), **BGG_financial.mod** (BGG)
- Results:

	NK	GK	BGG
LLs (2^{nd} stage)	5296.23	5300.27	5282.71
prob.	0.0173	0.9827	0.0000

Table: Marginal Log-likelihood Values and Posterior Model Odds

Limitations

- Such comparisons are important in the assessment of rival models
- A limitation is that the assessment of model fit is only relative to its other rivals with different restrictions
- The outperforming model in the space of competing models may still be poor (potentially misspecified)
- Ability of the absolute performance of one particular model against data
- Need to assess model's implied characteristics
- Model validation with by comparison with the second moments of data. This is often unsatisfactory.
- Use of endogenous priors as in Del Negro and Schorfheide (2008) and Christiano *et al.* (2011) helps.

Validation Based on Moment Criteria

- Ability to predict population moments (the absolute fit)
- Comparing second moments
  - Volatility - Standard Deviations
  - Co-Movement - Cross Correlations
  - Persistence - Autocorrelation
- To generate moments of endogenous variables in Dynare:
  - to get the model-generated moments based on actual data, simply use the *stoch_simul* keyword after the *estimation* command
  - useful options for the *estimation* command: *mode_file=FILENAME_mode*, *load_mh_file* and *nodiagnostic*
  - uses post-estimation solution (post. modes or means) of the model to produce various statistics of interest

Dynare Procedures

- This line of code simulates the model and generates moments of observables based on the post. distribution:

  stoch_simul(*OPTIONS*) y_obs c_obs i_obs er_obs pi_obs rn_obs etc.;

- This line of code reloads the computed posterior modes and simulated MH sequences:

  //estimation when modes and MCMC-MH draws already exist
  //filename_mode in the working directory
  estimation(*datafile* = *us_data*, *mode_compute*=**0**, *first_obs* = 12, *presample* = 11, **mode_file=FILENAME_mode**, *prefilter* = 1, *mh_replic*=**0**, *mh_nblocks* = 1, *mh_jscale* = 0.40, *mh_drop* = 0.2, *plot_priors* = 0, **load_mh_file**);

Results and Plots

- Again all simulation outputs are stored in the **FILENAME_results.mat** in the working directory $\Rightarrow$ reload it to extract useful information (in the structure array *oo_*)
- e.g. the simulated autocorrelation function can be found on the diagonal of the field *oo_.autocorr*
- Need subfunctions **acfcomp.m** and **autocov.m** to compute the sample ACF.
- In the working directory, **acfs_plot_.m** plots the sample ACFs and estimated ACFs from the model

Matching Moments: Estimated Parameter Means

Model	Output	Investments	Wage	Int. Rate	Inflation	Spread
			Means			
Data	0.3551	0.3635	0.3413	1.3318	0.8663	0.8031
NK	0.3544	0.3544	0.3544	1.3075	0.8490	0.0193
GK	0.3544	0.3544	0.3544	1.3318	0.8626	0.6900
BGG	0.3544	0.3544	0.3544	1.2956	0.8442	0.7307
			Standard Deviation			
Data	0.8017	2.1127	0.7531	0.9493	0.5855	0.5435
NK	0.8554	2.0730	1.1089	0.6966	0.5743	1.6902
GK	0.8274	2.2799	1.0869	0.7820	0.6031	2.8354
BGG	0.8344	2.1269	1.0964	0.7804	0.5960	1.8647
			Cross-correlation with Output			
Data	1.00	0.6791	0.0167	-0.0675	-0.1772	0.1211
NK	1.00	0.7128	0.4622	-0.1660	-0.1461	0.3249
GK	1.00	0.6555	0.4185	-0.1154	-0.05881	0.2502
BGG	1.00	0.6069	0.4273	-0.1837	-0.1919	0.2715

Autocorrelation Function Plots

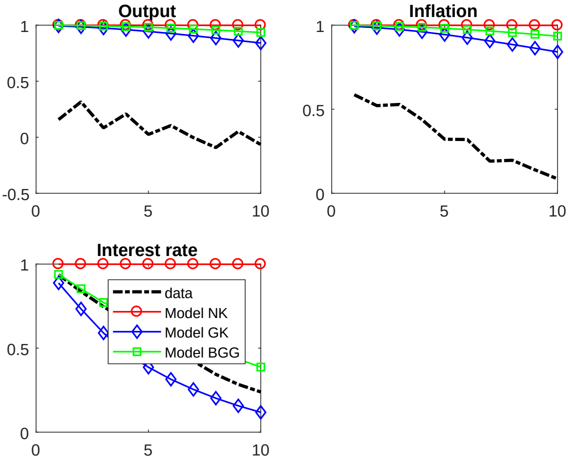


Figure: Autocorrelations of Observables in the Actual Data and in the Estimated Models

Exercise on Dynare: Estimation and Comparison

1. Consider the question: *How do the three models perform when additional data on the spread is used to constrain estimation?* To answer this use **usdata1947120173.mat** and the eight observed variables chosen in the notes, estimate the NK, GK and BGG models based on 10000 MCMC draws. Interpret the results. How important are wealth effects in each of these model?
2. You can answer the question by comparing the marginal likelihoods across the three models (with and without financial sectors). Use Matab file **modelcomparison.m** and compute the posterior model probabilities across these two models. What is the main conclusion?

Christiano, L. J., Trabandt, M., and Walentin, K. (2011). Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics and Control*, **35**(12), 1999–2041.

Del Negro, M. and Schorfheide, F. (2008). Forming Priors for DSGE models (and how it affects the Assessment of Nominal Rigidities. *Journal of Monetary Economics*, **55**(7), 1191–1208.

Fernandez-Villaverde, J., Rubio-Ramirez, J., Sargent, T., and Watson, M. W. (2007). ABC (and Ds) of Understanding VARs. *American Economic Review*, **97**(3), 1021–1026.

Levine, P., Pearlman, J., Wright, S., and Yang, B. (2019). Information, VARs and DSGE Models. Technical report, School of Economics Discussion Papers 1619, School of Economics, University of Surrey.