FINANCIAL FRICTIONS IN DSGE MODELS

The BGG Financial Accelerator Model

Paul Levine
Afrasiab Mirza

September 11, 2020

Introduction

- In a ‘costly state verification model’ due originally to Townsend (1979), the modelling strategy is once again to replace $\mathbb{E}_t[\Lambda_{t,t+1}R_{t+1}] = \mathbb{E}_t[\Lambda_{t,t+1}R_{k,t+1}]$ with a wedge that arises from the friction between a the risk neutral entrepreneur and a financial intermediary.
- The former borrow from the latter to purchase capital from capital producers at a price Q_t and combine it with labour through a production technology to produce wholesale output.
- In order to ensure they cannot grow out of the financial constraint, entrepreneurs exit with probability σ_E.
- As we shall see this setup introduces a wedge between the expected ex-ante riskless rate, $\mathbb{E}_t[R_{t+1}]$ and the expected return on capital $\mathbb{E}_t[R^K_{t+1}]$.

The Model

- The entrepreneur seeks loans l_t to bridge the gap between its net worth $n_{E,t}$ and the expenditure on new capital $Q_t k_t$, all end-of-period. Thus
$$l_t = Q_t k_t - n_{E,t} \tag{1}$$
where the entrepreneur's *real* net worth accumulates according to
$$n_{E,t} = R_t^K Q_{t-1} k_{t-1} - \frac{R_{l,t-1}}{\Pi_t} l_{t-1}$$
where R_t^K is the *real* return on capital as in the NK model and $R_{l,t}$ is the *nominal* loan rate to be decided in the contract.
- Note that all variables other than the loan rate are expressed in real terms.
- Model is not stationarized until we come to discussion of the Dynare code.

The Model

- In each period an idiosyncratic capital quality shock, ψ_t results in a return $R_t^K \psi_t$ which is the entrepreneur's private information.
- Default in period $t+1$ occurs when net worth becomes negative, i.e., when $n_{E,t+1} < 0$ and shock falls below a threshold $\bar{\psi}_{t+1}$ given by

$$\bar{\psi}_{t+1} = \frac{R_{l,t} l_t}{\Pi_{t+1} R_{t+1}^K Q_t k_t} \tag{2}$$

- With the idiosyncratic shock, ψ_t drawn from a density $f(\psi_t)$ with a lower bound ψ_{min}, the probability of default is then given by

$$p(\bar{\psi}) = \int_{\psi_{min}}^{\bar{\psi}_{t+1}} f(\psi) d\psi$$

The Model

- In the event of default the bank receives the assets of the firm and pays a proportion μ of monitoring costs to observe the realized return.
- Otherwise the bank receives the full payment on its loans, $R_{l,t+1} l_t$ where $R_{l,t}$ is the agreed loan rate at time t.
- At the heart of the model is the bank's *incentive compatibility (IC) constraint* at time $t+1$ given by

$$\mathbb{E}_t \left[(1-\mu) R^K_{t+1} Q_t k_t \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + (1 - p(\bar{\psi}_{t+1})) \frac{R_{l,t}}{\Pi_{t+1}} l_t \geq R_{t+1} l_t \right] \tag{3}$$

- The LHS of (3) is the expected return to the entrepreneur from the contract averaged over all realizations of the shock, the RHS is the return from a riskless bond.

The Model

- Eliminating the loan rate from (2), the IC constraint becomes

$$\mathbb{E}_t \left[R^K_{t+1} Q_t k_t \left((1-\mu) \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + \bar{\psi}_{t+1}(1 - p(\bar{\psi}_{t+1})) \right) \geq R_{t+1} l_t \right] \tag{4}$$

- Now define $\Gamma(\bar{\psi}_{t+1})$ to be the fraction of net capital received by the lender (the bank) and $\mu G(\bar{\psi}_{t+1})$ to be monitoring costs where

$$\Gamma(\bar{\psi}_{t+1}) \equiv \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + \bar{\psi}_{t+1}(1 - p(\bar{\psi}_{t+1})) \tag{5}$$

$$G(\bar{\psi}_{t+1}) \equiv \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi \tag{6}$$

The Model

- Then the optimal contract for the risk neutral entrepreneur solves

$$\max_{\bar{\psi}_{t+1},k_t} \mathbb{E}_t \left[\left(1 - \Gamma(\bar{\psi}_{t+1})\right) R^K_{t+1} Q_t k_t \right]$$

- given initial net worth $n_{E,t}$, subject to the IC constraint (4) which, using (1), (5) and (6), can be rewritten as

$$\mathbb{E}_t \left[R^K_{t+1} Q_t k_t \left[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}) \right] \geq R_{t+1}(Q_t k_t - n_{E,t}) \right]$$

The Model

- Let λ_t be the Lagrange multiplier associated with the IC constraint. Then the first order conditions are

$$
\begin{aligned}
k_t &: \quad \mathbb{E}_t\left[(1-\Gamma(\bar{\psi}_{t+1})R^K_{t+1} + \lambda_t\left[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))R^K_{t+1} - R_{t+1}\right]\right] = 0 \\
\bar{\psi}_{t+1} &: \quad \mathbb{E}_t\left[\Gamma'(\bar{\psi}_{t+1}) + \lambda_t(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))\right] = 0
\end{aligned}
$$

plus the binding IC condition if $\lambda_t > 0$ with $\lambda_t = 0$ if it does not bind.

- Combining these two conditions, we arrive at

$$
\mathbb{E}_t[R^K_{t+1}] = \mathbb{E}_t[\rho(\bar{\psi}_{t+1})R_{t+1}] \tag{7}
$$

where the *premium on external finance*, $\rho(\bar{\psi}_{t+1})$ is given by

$$
\rho(\bar{\psi}_{t+1}) = \frac{\Gamma'(\bar{\psi}_{t+1})}{\left[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1-\Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))\right]}
$$

The Model

- The *premium on external finance* equation which replaces the no arbitrage condition in the NK model is the crucial result coming out of the BGG model.
- Notice that in the limiting case as $\bar{\psi}_{t+1}$ and the probability of default tend to zero, and as monitoring costs μ disappear, $\Gamma \rightarrow 0$ and the risk premium $\rho(\bar{\psi}_{t+1}) \rightarrow 1$ returning us to the arbitrage condition in the NK model with a risk-neutral wholesale firm.

The Model

- So far we have set out the optimizing decision of the representative entrepreneur.
- We now aggregate assuming that entrepreneurs exit with fixed probability $1 - \sigma_E$.
- To allow new entrants start up we assume exiting entrepreneurs transfer a proportion ξ_E of their wealth to new entrants.
- Aggregate net worth then accumulates according to

$$N_{E,t} = (\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}_t))R_t^K Q_{t-1} K_{t-1}$$

- and the entrepreneur consumes

$$C_{E,t} = (1 - \sigma_E)(1 - \xi_E)(1 - \Gamma(\bar{\psi}_t))R_t^K Q_{t-1} K_{t-1}$$

The Model

- The resource constraint becomes

$$Y_t = C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t) R_t^K Q_{t-1} K_{t-1}$$

- The equilibrium is completed with the aggregate IC constraint

$$\mathbb{E}_t \left[R_{t+1}^K Q_t K_t \left[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}) \right] \right] = \mathbb{E}_t \left[R_{t+1} (Q_t K_t - N_{E,t}) \right]$$

which then helps pins down the contract rate $R_{l,t}$.

The Model

- Other post-recursive macroeconomic outcomes of interest are loans and the agreed loan interest rate given by

$$
\begin{aligned}
L_t &= Q_t K_t - N_{E,t} \\
\bar{\psi}_t &= \frac{R_{l,t-1} L_{t-1}}{R_t^K Q_{t-1} K_{t-1}} \frac{1}{\Pi_t} \\
R_t^K &= \frac{r_t^K + (1-\delta) Q_t}{Q_{t-1}} \\
r_t^K &= \frac{(1-\alpha) P_t^W Y_t^W}{P_t K_{t-1}}
\end{aligned}
$$

which completes the financial side of the model.

Summary of BGG Equilibrium

- The expected credit rate wedge is given by:

$$
\begin{aligned}
\mathbb{E}_t[R^K_{t+1}] &= \mathbb{E}_t[\rho(\bar{\psi}_{t+1})R_{t+1}] \\
N_{E,t} &= (\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}_t))R^K_t Q_{t-1}K_{t-1} \\
R^K_t Q_{t-1}K_{t-1}\left[\Gamma(\bar{\psi}_t) - \mu G(\bar{\psi}_t)\right] &= R_t(Q_{t-1}K_{t-1} - N_{E,t-1}) \text{ or} \\
\phi_{t-1}R^K_t[\Gamma(\bar{\psi}_t) - \mu G(\bar{\psi}_t)] &= R_t(\phi_{t-1} - 1)
\end{aligned}
$$

- where $\phi_t \equiv \frac{Q_t K_t}{N_{E,t}}$ is the leverage ratio and

$$
\rho(\bar{\psi}_{t+1}) = \frac{\Gamma'(\bar{\psi}_{t+1})}{\left[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1 - \Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))\right]}
$$

Summary of BGG Equilibrium

- with a resource constraint:

$$\begin{aligned} Y_t &= C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t))R_t^K Q_{t-1} K_{t-1} \\ C_{E,t} &= (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi}_t))R_t^K Q_{t-1} K_{t-1} \end{aligned}$$

- and post-recursive equations

$$\begin{aligned} L_t &= Q_t K_t - N_{E,t} \\ \bar{\psi}_t &= \frac{R_{l,t-1} L_{t-1}}{R_t^K Q_{t-1} K_{t-1}} \frac{1}{\Pi_t} \\ R_t^K &= \frac{r_t^K + (1-\delta) Q_t}{Q_{t-1}} \\ r_t^K &= \frac{(1-\alpha) P_t^W Y_t^W}{P_t K_{t-1}} \end{aligned}$$

Steady State

- The zero growth steady state is given by

$$\begin{aligned} R_k &= \rho(\bar{\psi})R \\ N_E &= (\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}))R_k QK \\ R_k QK\left[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})\right] &= R(QK - N_E) \text{ or} \\ R_k[\Gamma(\bar{\psi}) - \mu G(\bar{\psi})] &= R(\phi - 1) \end{aligned}$$

- where $\phi \equiv \frac{QK}{N_E}$ and

$$\rho(\bar{\psi}) = \frac{\Gamma'(\bar{\psi})}{\left[(\Gamma(\bar{\psi}) - \mu G(\bar{\psi}))\Gamma'(\bar{\psi}) + (1 - \Gamma(\bar{\psi}))(\Gamma'(\bar{\psi}) - \mu G'(\bar{\psi}))\right]}$$

Steady State

- with a resource constraint

$$\begin{aligned} Y &= C + C_E + G + I + \mu G(\bar{\psi}) R_k QK \\ C_E &= (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi})) R_k QK \end{aligned}$$

- and post-recursive equations

$$\begin{aligned} L &= QK - N_E \\ R_l &= \frac{\bar{\psi} R_k QK}{L} \\ R_k &= \frac{Z + (1-\delta)Q}{Q} \\ Z &= \frac{(1-\alpha)P^W Y^W}{K} \end{aligned}$$

Choice of Density function

- Our choice is a log-normal distribution.
- Thus $\psi_t = e^{x_t}$ where $x_t \sim N(-\frac{1}{2}\sigma_\psi^2, \sigma_\psi^2)$
- This guarantees that $\mathbb{E}[\psi_t] = 1$

Calibration

- The parameter values used in the NK model are as before.
- Note that, given the change in the resource constraint of the economy with respect to the NK case the steady state external file now solves for the steady state value of hours given the calibrated value of ϱ.
- Additional financial parameters to calibrate are A_{ψ}, σ_E, ξ_E and μ.
- These four parameters are calibrated to hit four targets:
  1. a default probability $p(\bar{\psi}) = 0.02$ (as in Faia and Monacelli (2007)),
  2. $\rho(\bar{\psi}) = 1.0025$ corresponding to a credit spread of 100 basis points as in GK,
  3. an entrepreneur leverage $\frac{QK}{N_E} = 2$ as in Bernanke *et al.* (1999) (rather lower than the bank leverage of 4 as in GK)
  4. and entrepreneurial consumption $\frac{C_E}{Y} = 0.075$.

Calibration

- With these values the calibrated parameters are:

Parameter	Calibrated Value
σ_ψ	0.3135
σ_E	0.9764
ξ_E	0.0067
μ	0.0284

Table: BGG Model with SW Preferences. Calibrated Parameters

- In the code we avoid a notational clash with μ already defined as a labour supply elasticity by denoting the monitoring cost parameter above as μ_{FF}.
- Compared with $\mu = 0.25$ in Faia and Monacelli (2007) we find small monitoring costs, a calibration necessary to hit our leverage target.

Dynare Code

- The mod file code for the BGG model is **BGG_normal_dist_SW.mod** set up with an external steady state that solves for the calibrated parameters above and for K and $\bar{\psi}$ using fsolve and an external steady state function.
- An analogous set-up is provided for SW preferences.

Exercises

1. Compare irfs for the NK, GK and BGG models with SW preferences.
2. In the BGG model now choose a *uniform distribution* for ψ and compare with the normal distribution.

Bernanke, B., Gertler, M., and Gilchrist, S. (1999). The Financial Accelerator in Quantitative Business Cycles. In M. Woodford and J. B. Taylor, editors, *Handbook of Macroeconomics, vol. 1C*. Amsterdam: Elsevier Science.

Faia, E. and Monacelli, T. (2007). Optimal interest rate rules, asset prices, and credit frictions. *Journal of Economic Dynamics and Control*, **31**(10), 3228 – 3254.

Townsend, R. M. (1979). Optimal Contracts and Competitive Markets with Costly State Verification. *Journal of Economic Theory*, **21**(2), 265 – 293.