FINANCIAL FRICTIONS IN DSGE MODELS

The Gertler-Kiyotaki (GK) Model

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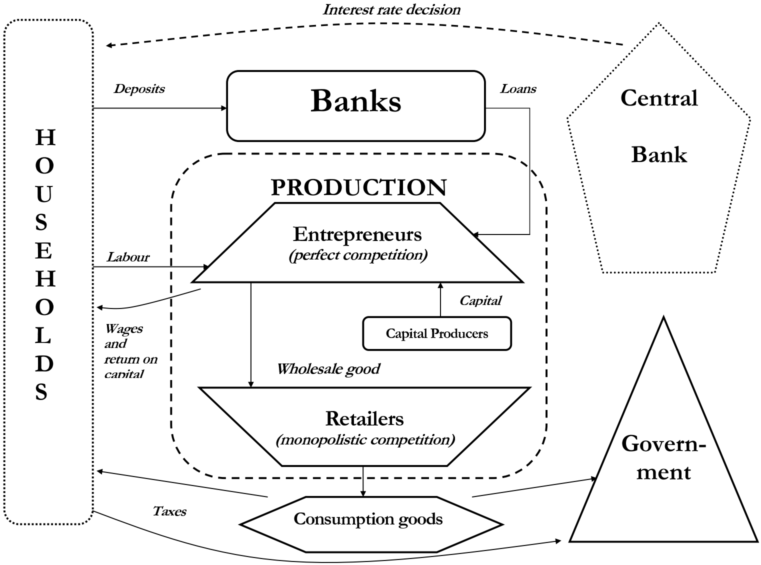
The NK Model

- Consists of **RBC core** with
  - External or internal habit in consumption options
  - SW preferences in consumption and hours
  - Cobb-Douglas production
- A **nominal side** consisting of
  - Price and wage stickiness in the form of staggered Calvo-type price setting
  - A nominal interest rate set by the CB in the form of simple Taylor rules with two options:
    - An 'implementable rule' that responds to the level and change of output relative to the deterministic steady state
    - A conventional Taylor rule that responds to the output gap (see later in Policy section)
- A non-zero steady state inflation and balanced growth with a stochastic trend
- Can add capacity utilization, fixed costs in wholesale production and Kimball aggregators (see Smets and Wouters (2007)).

Banking Model Interconnections



Two Approaches on the Course

We study two models of financial frictions (FF):

- The Financial Accelerator: 'Costly State Verification' - Bernanke *et al.* (1999) (BGG). The FF is between the bank and the firm.
- The Financial Accelerator: 'Costly Enforcement' - Gertler and Kiyotaki (2010a) (GK), Gertler *et al.* (2012), Gertler and Karadi (2011). The FF is now between the household and the bank.
- Default is also present in BGG.
- See the Notes for a more comprehensive survey

The Key Relationship

- In the NK model without financial frictions, **Expected discounted spread =0**
- That is $\underbrace{1 = E_t[\Lambda_{t,t+1} R_{t+1}]}_{\text{Euler Consumption Eqn}} = E_t[\Lambda_{t,t+1} R^K_{t+1}]$

where $\Lambda_{t,t+1} = \frac{\beta U_{C,t+1}}{U_{C,t}}$ is the $[t, t+1]$ stochastic discount factor and the *gross* return on capital is given by

$$R^K_t = \frac{r^K_t + (1-\delta)Q_t}{Q_{t-1}}$$

where the rental rate equated with the MPK in the wholesale sector is

$$r^K_t = (1-\alpha)\frac{P^W_t}{P_t}\frac{Y^W_t}{K_{t-1}/(1+g_t)}$$

(assuming CD technology) and the ex post real interest rate is

$$R_t = \frac{R_{n,t-1}}{\Pi_t}$$

The GK Model with Financial Frictions

- Replace $E_t[\Lambda_{t,t+1} R_{t+1}] = E_t[\Lambda_{t,t+1} R^K_{t+1}]$ with a banking sector that introduces a wedge between these expected returns
- Given a certain deposit level, a bank can lend frictionlessly to non-financial firms against their future profits.
- The friction arises between the household and the bank
- The activity of the bank can be summarized in two phases.
  1. Banks raise deposits from households.
  2. Banks uses the deposits to make loans to firms.

Banking Sequence of Events

1. Banks raise deposits, d_t from households at a real deposit net rate R_t over the interval $[t-1, t]$
2. Banks make loans to firms.
3. Loans are s_t at a price Q_t. s_t is the number of claims to one unit of firms' capital, so the asset against which the loans are obtained is end-of-period capital K_t. Capital depreciates at a rate δ in each period.

Bank Balance Sheet and Net Worth Accumulation

- $Q_t s_t = n_t + d_t$, where LHS is assets, RHS liabilities.
- $n_t(1+g_t) = n_{t-1} + (R_t^K - 1)Q_{t-1}s_{t-1} - (R_t - 1)d_{t-1} = R_t^K Q_{t-1}s_{t-1} - R_t d_{t-1}$ is net worth.
- Thus $Q_t s_t(1+g_t) + R_t d_{t-1} = R_t^K Q_{t-1}s_{t-1} + d_t(1+g_t)$ is the bank's budget constraint.
- Also $n_t = R_t n_{t-1}/(1+g_t) + (R_t^K - R_t)Q_{t-1}s_{t-1}/(1+g_t)$. Net worth at the end of period t equals the gross return at the real rate plus the excess return over the latter on the assets
- In the RBC model of GK, R_t is riskless, but in our NK model with inflation it the risky ex post real interest rate.
- In a richer model inter-bank lending and outside equity can be added to the balance sheet.

The Banker's Objective

- There is a consolidated households of bankers and workers. If bankers lasted for ever the financial constraint would eventually cease to bind.
- Banks exit and become workers with probability $1 - \sigma_B$ per period, Workers become banks with the same probability keeping proportions fixed.
- The banker's objective in GK is at the end of period t to maximize expected terminal wealth

$$V_t = E_t \sum_{i=1}^{\infty} (1 - \sigma_B) \sigma_B^{i-1} \Lambda_{t,t+i} n_{t+i}$$

  where $\Lambda_{t,t+i}$ is the $[t, t+i]$ stochastic discount factor corresponding to the consumer's optimization problem.
- If we allowed the two groups to be distinct agents we could introduce differing appetites for risk as in Wickens (2011).

Endogenous Constraint on the Banks

- After a bank obtains funds, the banks manager may transfer a fraction of assets to her family.
- Households therefore limit the funds they lend to banks.
- In order to ensure that bankers do not divert funds the following incentive constraint must hold:

$$V_t \geq \Theta_B Q_t s_t \tag{1}$$

  where $1 - \Theta_B$ is the fraction of funds that can be reclaimed by creditors. Thus for households to be willing to supply funds, the banks franchise value V_t must be at least as large as its gain from diverting funds.
- Assume constraint is either always binding or absent as in basic NK. Current research considers *an occasionally binding constraint*.

Solution of Banker's Problem

- The solution procedure of Gertler and Kiyotaki (2010b), included in Appendix 3, can be made far simpler, as we show here.
- Assume that $V_t(n_{t+1}) = E_t[\Lambda_{t,t+1}\Omega_{t+1}n_{t+1}]$
- Ω_t is shadow value of a unit of net worth.
- Write the Bellman equation for a given path for n_t as $V_{t-1}(n_t) = E_{t-1}\Lambda_{t-1,t}[(1-\sigma_B)n_t + \sigma_B \max_{s_t} V_t(n_{t+1})]$
- Thus we need to maximize $V_t = E_t\Lambda_{t,t+1}\Omega_{t+1}n_{t+1} = E_t\Lambda_{t,t+1}\Omega_{t+1}[(R^K_{t+1} - R_{t+1})Q_t s_t + R_{t+1}n_t]$ subject to the constraint $V_t \geq \Theta_B Q_t s_t$.
- When the constraint is binding – which is when $\Theta_B > \mathbb{E}_t[\Omega_{t+1}\Lambda_{t,t+1}(R^K_{t+1} - R_{t+1})]$ – this yields $Q_t s_t = \frac{\mathbb{E}_t[\Omega_{t+1}\Lambda_{t,t+1}R_{t+1}]}{\Theta_B - \mathbb{E}_t[\Omega_{t+1}\Lambda_{t,t+1}(R^K_{t+1} - R_{t+1})]} n_t$

Solution of Banker's Problem (cont)

- Hence we infer that $\frac{Q_t s_t}{n_t} = \phi_t = \frac{\mathbb{E}_t[\Omega_{t+1}\Lambda_{t,t+1}R_{t+1}]}{\Theta - \mathbb{E}_t[\Omega_{t+1}\Lambda_{t,t+1}(R^K_{t+1} - R_{t+1})]}$
- In addition, substituting the optimal value of $Q_t s_t$ into V_t and then computing the value of V_{t-1}, as in the previous slide, yields $\Omega_t = 1 - \sigma_B + \sigma_B \Theta_B \phi_t$
- Ω_t, the shadow value of a unit of net worth, is as obtained in the original GK solution

Aggregation

- Aggregating up to N_t etc, accounting for banks that quit and enter, the balance sheet and aggregate leverage are:

$$
\begin{aligned}
Q_t S_t &= N_t + D_t \\
\phi_t &= \frac{Q_t S_t}{N_t}
\end{aligned}
$$

- At the aggregate level net worth is the sum of existing (old) bankers and new bankers:

$$N_t = N_{o,t} + N_{n,t}$$

where

$$(1 + g_t) N_{o,t} = \sigma_B \{ (r_t^K + (1 - \delta) Q_t) S_{t-1} - R_t D_{t-1} \}$$

- To allow new bankers to operate with some net worth, we assume that the family transfers to each one a fraction ξ_B of the value value of assets of the exiting bank implying:

$$(1 + g_t) N_{n,t} = \xi_B [r_t^K + (1 - \delta) Q_t] S_{t-1}$$

- This completes the banking model.

Summary

- Aggregating up to N_t etc, accounting for banks that quit and enter – the latter beginning operation with a net worth transferred as a fraction ξ_B of the assets of exiting banks.
- Now an *expected spread* emerges if the IC constraint binds: $\mu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} (R^K_{t+1} - R_{t+1}) > 0$ where μ_t is given by

$$\mu_t = \Theta_B - \mathbb{E}_t[\Omega_{t+1} \Lambda_{t,t+1} R_{t+1}]/\phi_t$$

- Given K_t, aggregate net worth accumulates according to

$$\begin{aligned} (1+g_t)N_t &= R^K_t(\sigma_B + \xi_B) Q_{t-1} S_{t-1} - \sigma_B R_t D_{t-1} \\ D_t &= Q_t S_t - N_t \\ S_t &= K_t \end{aligned}$$

The GK Model with Outside Equity

The model, which closely follows Gertler *et al.* (2012) - henceforth GKQ - adds an extra ingredient, the option to raise funds by issuing equity as well as household deposits. Now we have the following sequence of events:

1. Banks raise deposits, d_t, *and outside equity,* e_t, from households at a real deposit net rate R_{t+1} and *equity net rate* R^E_{t+1} respectively over the interval $[t, t+1]$, the 'time period t'.
2. Banks make loans to firms.
3. Loans are s_t at a price Q_t. The asset against which the loans are obtained is end-of-period capital K_t. Capital depreciates at a rate δ in each period. *The price of outside equity is* $q_t > Q_t$ in our model with financial constraints.

The Banker's Optimization Problem

- The banking sector's balance sheet of the form:

$$Q_t s_t = n_t + q_t e_t + d_t \tag{2}$$

- Net worth of the bank accumulates according to:

$$n_t = R_t^K Q_{t-1} s_{t-1} - R_t d_{t-1} - R_t^E q_{t-1} e_{t-1} \tag{3}$$

where real returns on bank assets and equity are given by

$$\begin{aligned} R_t^K &= \frac{[r_t^K + (1-\delta)Q_t]}{Q_{t-1}} \\ R_t^E &= \frac{[r_t^K + (1-\delta)q_t]}{q_{t-1}} \end{aligned}$$

- As before the banker's objective is to maximize, subject to (2) and (3) and a borrowing contraint, the expected discounted terminal wealth

$$V_t = \mathbb{E}_t \sum_{i=1}^{\infty} (1-\sigma_B)\sigma_B^i \Lambda_{t,t+i} n_{t+i}$$

The Borrowing Constraint

- The borrowing constraint is now

$$V_t \geq \Theta(x_t) Q_t s_t \tag{4}$$

where $x_t \equiv \frac{q_t e_t}{Q_t s_t}$ is the fraction of bank assets financed by outside equity.

- $\Theta'_t >, \Theta''_t > 0$ captures the idea that it is easier to divert assets funded by outside equity than by households.
- As before, the incentive constraint states that for households to be willing to supply funds to a bank, the bank's franchise value V_t must be at least as large as its gain from diverting funds.
- For the function $\Theta(x_t)$ we choose

$$\Theta_t \equiv \Theta(x_t) = \theta(1 + \epsilon x_t + \kappa x_t^2/2)\,;\ \theta,\ \kappa > 0,\ \epsilon < 0 \tag{5}$$

Solution of the Banker's Problem

- To solve the problem we look for a linear solution of the form:

$$V_t = E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1}$$

- Proceeding as before and aggregating we arrive at the solution:

$$\begin{aligned}
S_t &= K_t \\
x_t &= \frac{q_t E_t}{Q_t S_t} \\
Q_t S_t &= \phi_t N_t \\
\phi_t &= \frac{\mathbb{E}_t[\Omega_{t+1}\Lambda_{t,t+1}R_{t+1}]}{\Theta(x_t) - \mathbb{E}_t[\Omega_{t+1}\Lambda_{t,t+1}(R^K_{t+1} - R_{t+1} + (R_{t+1} - R^E_{t+1})x_t)]} \\
N_t &= ([r^K_t + (1-\delta)Q_t](\sigma_B + \xi_B)S_{t-1} \\
&\quad -\sigma_B(r^K_t + (1-\delta)q_t)E_{t-1} - \sigma_B R_t D_{t-1})/(1+g_t) \\
D_t &= Q_t S_t - N_t - q_t E_t \\
\frac{\Theta'(x_t)}{\Theta(x_t)} &= \frac{\mathbb{E}_t \Lambda_{t,t+1}(R_{t+1} - R^E_{t+1})}{\mathbb{E}_t(\Lambda_{t,t+1}(R^K_{t+1} - R_{t+1} + (R_{t+1} - R^E_{t+1})x_t))}
\end{aligned}$$

Solution of the Banker's Problem (continued)

- where Ω_t is given by

$$\Omega_t \;\; = \;\; 1 - \sigma_B + \sigma_B \Theta_t \phi_t$$

- The missing equation to close the model is closed is the household arbitrage condition

$$\mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}] = \mathbb{E}_t[\Lambda_{t,t+1} R^E_{t+1}] = 1$$

  using the Euler consumption equation.
- Note that this last condition implies that in steady state $R^E = R$ and therefore $\Theta'(x) = 0$.

Calibration of GK Model with Outside Equity

- The steady state is solved for K with easy recursive structure
- For the function $\Theta(x_t)$ we choose

$$\Theta_t \equiv \Theta(x_t) = \theta_{FF}(1 + \epsilon x_t + \kappa_{FF} x_t^2/2)$$

- Since in the steady state $R = R^E$, $\mu_e = \Theta' = 0$ - see first two equations of previous slide. We then obtain

$$x = -\frac{\epsilon}{\kappa_{FF}}$$

- We set $\epsilon = -2$ (as in GK). Then by choosing a target for x we can pin down the remaining parameter κ.
- Using 'low risk' and 'high risk' scenarios, Gertler *et al.* (2012)) use a *risky steady state* which can be solved analytically to pin down ϵ. Our stochastic steady states are *ergodic means* throughout.
- Parameters θ_{FF}, κ_{FF} and ξ_B are calibrated to hit a *total* leverage target $\frac{QS}{N+qE} = 4$, a spread $R^K - R = 0.01/4$. This gives a calibration shown in Table 1 below.

Calibrated Parameters

Parameter	Calibrated Value
θ_{FF}	0.4274
κ_{FF}	13.333
ξ_B	0.0023
ϵ	-2

Table: GK-equity Model with Internal Habit. Calibrated Parameters.

Steady States of NK and GK: External Habit

- The deterministic steady state is solved in an external ss matlab program using fsolve. Given K it is very easy to find a recursive form.
- We can compare the deterministic steady states of NK and GK

Variable	NK	GK
C	0.5140	0.5130
Y	0.8347	0.8117
H	0.3300	0.3300
H_d	0.3293	0.33293
θ	2.640	2.643
$Welfare$	-379.07	-379.67
$CEequiv$	4.736	4.703
ce	0	0.127

- ce is the consumption % equivalent relative to NK given by $ce = (Welfare(NK) - Welfare(GK))/CEequiv$ (see policy section later) .
- Now compare with internal habit

Steady states of NK and GK: Internal Habit

Variable	NK	GK
C	0.5184	0.5130
Y	0.8347	0.8117
H	0.3300	0.3300
H_d	0.3293	0.33293
θ	3.051	3.057
$Welfare$	-284.31	-285.89
$CEequiv$	2.664	2.665
ce	0	0.593

- Thus the welfare cost of financial frictions are far higher with internal habit in the steady state! Why?
- The reason is that consumption is a negative externality with external Habit. With suitable parameter values financial frictions by lowering consumption can even be welfare-enhancing!

Capital Quality Shocks

- Following the macro-finance literature, in all our banking and NK models we add a capital quality shock, say KQ_{t+1}, that wipes out or enhances capital available in period t going into period $t+1$.
- $S_t = [(1-\delta)K_{t-1}/(1+g_t) + (1-S(X_t))I_t]$ is now 'capital in process' which is transformed by the production process into capital for next period's production according to $K_t = KQ_{t+1}S_t$.
- Thus capital in process evolves according to

$$S_t = (1-\delta)KQ_t S_{t-1}/(1+g_t) + (1-S(X_t))I_t \tag{6}$$

- Capital quality shock also affects the balance sheet of the banks. Now net returns are given by

$$R_t^K = KQ_t \frac{r_t^K + (1-\delta)Q_t}{Q_{t-1}}$$

- It follows from (6) and $K_t = KQ_{t+1}S_t$ that

$$K_t = KQ_{t+1}((1-\delta)K_{t-1}/(1+g_t) + (1-S(X_t))I_t)$$

Dynare Software

- The GK banking models for SW preferences, **GK_SW.mod**, are in folder **GK**
- External steady state matlab files **GK_SW_steadystate.m** then call function **ss_fun_GK_SW.m**. fsolve then solves for the steady state K and performs the calibration.
- A matlab file **graphs_irfs_compare_NK_GK_BGG.m** to compare the irfs of the NK and banking models is also included in the folder.

Exercises

1. Add the capital quality shock KQ_t to the GK model with equity and SW preferences. Use the graph plotter to compare KQ_t and IS_t shocks.

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