FINANCIAL FRICTIONS IN DSGE MODELS

The NK Model

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From RBC to NK

Recall The RBC Core

- Households, with external habit or internal in their JR utility, make an intertemporal utility-maximizing choice of consumption and labour supply subject to a budget constraint.
- Net assets consists of capital employed by firms
- Firms (wholesale=retail) produce output according to a crt production technology and choose labour and capital inputs to minimize cost
- Investment adjustment costs and capital producers
- Labour, output and financial markets clear
- A balanced exogenous non-zero growth steady state
- A nominal side with monetary policy that has no real effects
- Now add nominal price and wage frictions

Two NK Models

- Two models are commonly found in the literaure

  1. The "workhorse" NK model with labour as the only factor of production. Prices are sticky, but wages flexible. Suitable for analytical results - see Woodford (2003) for example.
  2. The model of Smets and Wouters (2007), henceforth SW with capital, sticky wages, capacity utilization and fixed costs.

- We will study a slimmed down version of the SW model without capacity utilization, fixed costs and **Dixit-Stiglitz** (DS) rather than **Kimball Preferences** (KP).
- DS preferences are only consistent with an empirically implausible low frequency of price changes (a high slope of the Phillip's Curve).
- But KP require an empirically implausible value of the price super-elasticity to have an impact.
- See Deak *et al.* (2020) for details.

Our New Keynesian Model

- Our NK then consists of an RBC real core with JR or SW utility, labour and capital as factors of production, external habit in consumption.
- A nominal side consisting of
  - Prices set by the retail sector and nominal wages by a trade union
  - A nominal interest rate set by the policymaker (already in our RBC model)
- We first introduce *price stickiness* in the form of staggered Calvo-type price setting in the retail sector

Illustration of NK Model

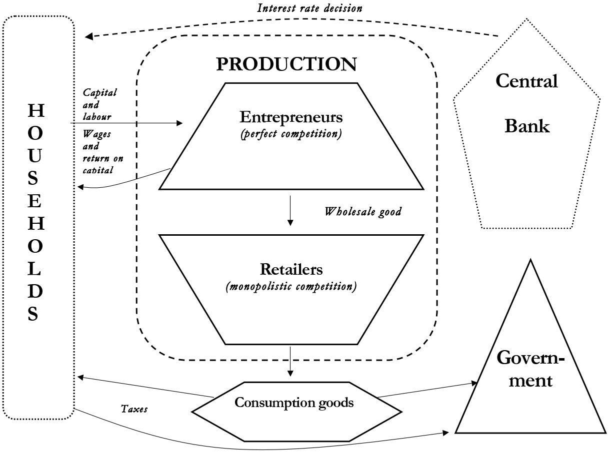

Figure: **NK Model**

SW Preferences

- As with CD preferences, those of SW are compatible with balanced growth (see King *et al.* (1988)).
- In stationarized form, with external habit in consumption, household j has a single-period *non-separable* utility

$$U_t^j = \frac{(C_t^j - \chi C_{t-1}/(1+g_t))^{1-\sigma_c} \exp\left(\frac{(\sigma_c-1)(H_t^j)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c}; \quad \chi \in [0,1)$$

$$\to \log(C_t^j - \chi C_{t-1}/(1+g_t)) + \frac{(H_t^j)^{1+\sigma_l}}{1+\sigma_l} \text{ as } \sigma_c \to 1$$

where C_{t-1} is aggregate per capita consumption

- Whereas with internal habit we have

$$U_t^j = \frac{(C_t^j - \chi C_{t-1}^j/(1+g_t))^{1-\sigma_c} \exp\left(\frac{(\sigma_c-1)(H_t^j)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c}; \quad \chi \in [0,1)$$

$$\to \log(C_t^j - \chi C_{t-1}^j/(1+g_t)) + \frac{(H_t^j)^{1+\sigma_l}}{1+\sigma_l} \text{ as } \sigma_c \to 1$$

SW Preferences

- Defining an instantaneous marginal utility by

$$U_{C,t} = (C_t - \chi C_{t-1}/(1+g_t))^{-\sigma_c} \exp\left(\frac{(\sigma_c - 1)H_t^{1+\sigma_l}}{1+\sigma_l}\right)$$

- Then in a symmetric equilibrium, household first-order conditions are

$$1 = \mathbb{E}_t\left[R_{t+1}\Lambda_{t,t+1}\right] \text{ where } \Lambda_{t,t+1} \equiv \beta_{g,t+1}\frac{\lambda_{t+1}}{\lambda_t}$$

$$U_{H,t} = -H_t^{\sigma_l}(C_t - \chi C_{t-1}/(1+g_t))^{1-\sigma_c} \exp\left(\frac{(\sigma_c - 1)H_t^{1+\sigma_l}}{1+\sigma_l}\right)$$

$$\frac{U_{H,t}}{\lambda_t} = -W_t$$

where for external habit and internal habit respectively we have

$$\lambda_t = U_{C,t}$$
$$\lambda_t = U_{C,t} - \beta\chi\mathbb{E}_t[U_{C,t+1}]$$

SW Preferences

- Note that unless $\sigma_c \to 1$, the *separable* utility function:

$$U_t^j = \frac{(C_t^j - \chi C_{t-1}/(1+g_t))^{1-\sigma_c} - 1}{1-\sigma_c} + \frac{(H_t^j)^{1+\sigma_l}}{1+\sigma_l}$$

  in Woodford (2003) is *not* compatible with balanced growth.

- Parameter σ_l is referred to by Smets and Wouters (2007) as the labour supply elasticity.

The New Keynesian Phillips Curve

- Introduce a retail sector producing differentiated goods under monopolistic competition. This sector converts homogeneous output from a competitive wholesale sector. The aggregate prices in the two sectors are given by P_t and P_t^W respectively and $P_t > P_t^W$ from the *mark-up* possible under monopolistic competition
- The *real marginal cost* of producing each differentiated good $MC_t \equiv \frac{P_t^W}{P_t}$. In the RBC model $P_t = P_t^W$ so $MC_t = 1$ and the *marginal cost is constant*.
- In the NK model retailers are locked into price-contracts and cannot their prices every period. Their marginal costs therefore vary. In periods of high demand they simply increase output until they are able to change prices. Then equating the real wage with the marginal product in the wholesale sector gives $W_t = \frac{MC_t Y_t^W}{H_t}$ where Y_t^W is wholesale output.

The NK Model: Dixit-Stiglitz Aggregators

- Following Dixit and Stiglitz (1977) assume *monopolistic competition* in the retail sector that converts a homogeneous wholesale good into differentiated goods.
- Then we can model *price-setting* by firms.
- Define *Dixit-Stiglitz CES aggregators* for aggregate consumption and price over differentiated goods m by

$$C_t = \left(\int_0^1 C_t(m)^{(\zeta-1)/\zeta} dm \right)^{\zeta/(\zeta-1)}$$

$$P_t = \left[\int_0^1 P_t(m)^{1-\zeta} dm \right]^{\frac{1}{1-\zeta}}$$

- With these definitions we have that

$$P_t C_t = \text{aggregate expenditure}$$

The NK Model: Demand for Differentiated Goods

- The parameter ζ turns out to be a price-elasticity.
- Dixit-Stiglitz aggregators lead to a demand for consumption of good m is given by

$$C_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta} C_t \tag{1}$$

- Similarly for investment and government goods so in aggregate

$$Y_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta} Y_t \tag{2}$$

- Now we can model *price stickiness*

The NK Model: Calvo Price Contracts

- There is a probability of $1-\xi_p$ at each period that the price of each retail good m is set optimally to $P_t^0(m)$; otherwise it is held fixed.
- For each retail producer m, given its real marginal cost MC_t, the objective is at time t to choose $\{P_t^0(m)\}$ to maximize discounted real profits

$$E_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_t^0(m) - P_{t+k} MC_{t+k}\right]$$

subject to

$$Y_{t+k}(m) = \left(\frac{P_t^O(m)}{P_{t+k}}\right)^{-\zeta} Y_{t+k} \qquad (3)$$

where $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the (non-stationarized) stochastic discount factor over the interval $[t, t+k]$.

The NK Model: Calvo Price Contracts

- The solution to this optimization problem is

$$E_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_t^0(m) - \frac{1}{(1-1/\zeta)} P_{t+k} MC_{t+k} \right] = 0$$

$$P_{t+1}^{1-\zeta} = \xi_p P_t^{1-\zeta} + (1-\xi_p)(P_{t+1}^0)^{1-\zeta} \quad (4)$$

- Using (3) and rearranging this leads to

$$P_t^O = \frac{1}{(1-1/\zeta)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} (P_{t+k})^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} (P_{t+k})^{\zeta} Y_{t+k}} \quad (5)$$

where the m index is dropped as all firms face the same marginal cost so the right-hand side of the equation is independent of firm size or price history.

The NK Model: Calvo Price Contracts

- Now define k periods ahead inflation as

$$\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t}$$

- To ease the notation in what follows we denote $\Pi_t = \Pi_{t-1,t}$ and $\Pi_{t+1} = \Pi_{t,t+1}$. We can now write the fraction (5)

$$\frac{P_t^O}{P_t} = \frac{1}{(1-1/\zeta)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} \left(\Pi_{t,t+k}\right)^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} \left(\Pi_{t,t+k}\right)^{\zeta-1} Y_{t+k}} \tag{6}$$

- And (4) can be written

$$1 = \xi_p \left(\Pi_t\right)^{\zeta-1} + (1-\xi_p) \left(\frac{P_t^O}{P_t}\right)^{1-\zeta} \tag{7}$$

The NK Model: Price Dynamics

Details of algebra in Appendix 2 of the Notes

- **Price dynamics** as difference equations: Using the Lemma in Section 5, we can express the foc above as difference equations:

$$\frac{P_t^0}{P_t} = \frac{J_t^p}{JJ_t^p}$$

$$JJ_t^p - \xi_p E_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta-1} JJ_{t+1}^p] = Y_t$$

$$J_t^p - \xi_p E_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta} J_{t+1}^p] = \left(\frac{1}{1-\frac{1}{\zeta}}\right) Y_t MC_t MS_t$$

$$\Pi_t : \quad 1 = \xi_p \Pi_t^{\zeta-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{1-\zeta}$$

- Note we have added a *mark-up shock* MS_t to marginal costs.
- **Real marginal costs** are no longer fixed and are given by

$$MC_t = \frac{P_t^W}{P_t} = \frac{W_t}{F_{H,t}} = \frac{W_t H_t}{\alpha Y_t^W}$$

The NK Model: Monetary Policy

- The nominal interest rate is given by the following Taylor-type rules

$$\textbf{Implementable}: \log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right)$$
$$+ \quad (1-\rho_r)\left[\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right)\right] + \log MPS_t \quad \textbf{or}$$
$$\textbf{Conventional Taylor}: \log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right)$$
$$+ \quad (1-\rho_r)\left[\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y_t^F}\right)\right] + \log MPS_t$$

  where Y_t^F is the flexi-price level of output (as in RBC model) and MPS_t is a monetary policy shock process.
- The 'implementable' form stabilizes output about its steady state, the conventional form about Y_t^F. Then θ_π and θ_y are the long-run elasticities of the inflation and output respectively with respect to the interest rate. The "Taylor principle" requires $\theta_\pi > 1$.

The NK Model: Price Dispersion

See Appendix 2 of Notes

- The remaining change is that **price dispersion** Δ_t^p reduces output which, as shown in Appendix 2.3 of Notes , is now given by

$$Y_t = \frac{Y_t^p}{\Delta_t^p}$$

$$\Delta_t^p \equiv \frac{1}{n}\sum_{j=1}^{n}(P_t(j)/P_t)^{-\zeta} = \xi_p \Pi_t^{\zeta} \Delta_{t-1}^p + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{-\zeta}$$

- If steady state net inflation is zero, Δ_t^p is of second order so for a first-order approximation can be ignored.

Exogenous Shock Processes

- The model is closed with seven exogenous AR1 shock processes to technology, government spending, the real marginal cost (the latter being interpreted as a mark-up shock), the marginal rate of substitution, an investment shock, a risk premium shock and a shock to monetary policy

$$
\begin{aligned}
\log A_t - \log A &= \rho_A(\log A_{t-1} - \log A) + \epsilon_{A,t} \\
\log G_t - \log G &= \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t} \\
\log MS_t - \log MS &= \rho_{MS}(\log MS_{t-1} - \log MS) + \epsilon_{MS,t} \\
\log MRSS_t - \log MRSS &= \rho_{MRSS}(\log MRSS_{t-1} - \log MRSS) + \epsilon_{MRSS} \\
\log IS_t - \log IS &= \rho_{IS}(\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \\
\log RPS_t - \log RPS &= \rho_{RPS}(\log RPS_{t-1} - \log RPS) + \epsilon_{RPS,t} \\
\log MPS_t - \log MPS &= \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t}
\end{aligned}
$$

- Other shocks are possible such as to preferences and capital quality.

The NK Model Steady State: Real Component

Consider first the case of a zero net inflation and growth steady state ($\Pi = 1$, $g = 0$). Then the *real* component of the steady state is that of the RBC model with a wholesale-retail mark-up $\frac{P^W}{P}$:

$$
\begin{aligned}
Q &= 1 \text{ (because } S(X) = S'(X) = S(0) = S'(0) = 0) \\
R &= R_n = \frac{1}{\beta} \\
Y^W &= (AH)^{\alpha} K^{1-\alpha} \\
\Delta^p &= 1 \\
Y &= \frac{Y^W}{\Delta^p} = Y^W \text{ since } \Delta = 1 \\
\frac{\alpha P^W}{P} &= W \\
\frac{PK}{P^W Y^W} &= \frac{1-\alpha}{R-1+\delta} \\
I &= \delta K
\end{aligned}
$$

The NK Model Steady State: Nominal Component

- For a non-zero steady state net inflation rate (gross rate $\Pi > 1$):

$$
\begin{aligned}
R_n &= \Pi R \\
J^p(1-\beta\xi_p\Pi^{\zeta}) &= \frac{YMC}{\left(1-\frac{1}{\zeta}\right)} \\
JJ^p(1-\beta\xi_p\Pi^{\zeta-1}) &= Y \\
\frac{J^p}{JJ^p} &= \left(\frac{1-\xi_p\Pi^{\zeta-1}}{1-\xi_p}\right)^{\frac{1}{1-\zeta}} \\
MC &= \left(1-\frac{1}{\zeta}\right)\frac{J^p(1-\beta\xi_p\Pi^{\zeta})}{JJ^p(1-\beta\xi_p\Pi^{\zeta-1})} \\
\Delta^p &= \frac{(1-\xi_p)^{\frac{1}{1-\zeta}}(1-\xi_p\Pi^{\zeta-1})^{\frac{-\zeta}{1-\zeta}}}{1-\xi_p\Pi^{\zeta}}
\end{aligned}
$$

- For a zero-inflation steady state $\Pi = 1$ we arrive $\frac{J}{JJ} = \Delta = 1$ and $MC = \left(1-\frac{1}{\zeta}\right)$. Note there is a long-run inflation-output trade-off.

NK Model with Indexation

- Price Indexation introduces inflation persistence in an ad hoc fashion
- With indexing by an amount γ_p the optimal price-setting first-order condition for a firm j setting a new optimized price $P_t^0(j)$ becomes

$$P_t^0(j)E_t\left[\sum_{k=0}^{\infty}\xi_p^k\frac{\Lambda_{t,t+k}}{P_{t+k}}Y_{t+k}(m)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p}\right] = \frac{1}{(1-1/\zeta)}E_t\left[\sum_{k=0}^{\infty}\xi_p^k\Lambda_{t,t+k}MC_{t+k}Y_{t+k}(m)\right]$$

- Price dynamics are now given by

$$\frac{P_t^0}{P_t} = \frac{J_t^p}{JJ_t^p}$$

$$JJ_t^p - \xi_p E_t[\Lambda_{t,t+k}\tilde{\Pi}_{t+1}^{\zeta-1}JJ_{t+1}^p] = Y_t \text{ where } \tilde{\Pi}_t \equiv \frac{\Pi_t}{\Pi_{t-1}^{\gamma_p}}$$

$$J_t^p - \xi_p E_t[\Lambda_{t,t+k}\tilde{\Pi}_{t+1}^{\zeta}J_{t+1}^p] = \frac{1}{1-\frac{1}{\zeta}}MC_tY_t$$

Indexing in the Steady State

- An alternative model of indexing assumes that prices are indexed to a weighted average of last period and trend (steady state) inflation.
- If we denote the two weights by γ_p and $\bar{\gamma}_p$ then the previous dynamics replaces $\tilde{\Pi}_t$ above with

$$\tilde{\Pi}_t \equiv \frac{\Pi_t}{\Pi_{t-1}^{\gamma_p} \Pi^{1-\bar{\gamma}_p}} \tag{8}$$

- In Smets and Wouters (2007) it is assumed that $\bar{\gamma}_p = \gamma_p$ so that $\tilde{\Pi} = 1$ in the steady state which eliminates the effect of state-state inflation in the equilibrium.
- In the coding of our model we could allow for options $\bar{\gamma}_p = \gamma_p$ and $\bar{\gamma}_p = 0$, but the SW case is not working.

Wage Stickiness

- To introduce wage stickiness we now assume that each household supplies homogeneous labour at a nominal wage rate $W_{h,t}$ to a monopolistic trade-union
- She then differentiates the labour and sells type $H_t(j)$ at a nominal wage $W_{n,t}(j) > W_{h,t}$ to a labour packer in a sequence of Calvo staggered nominal wage contracts.
- The real wage is then defined as $W_t \equiv \frac{W_{n,t}}{P_t}$. We now have to distinguish between *price inflation* which now uses the notation $\Pi_t^p \equiv \frac{P_t}{P_{t-1}}$ and *wage inflation*. $\Pi_t^w \equiv \frac{W_{n,t}}{W_{n,t-1}}$
- As with price contracts we employ Dixit-Stiglitz quantity and price aggregators. Calvo probabilities are now ξ_p and ξ_w for price and wage contracts respectively. Similarly J_t^p, JJ_t^p and J_t^w, JJ_t^w drive price and nominal wage dynamics respectively with dispersions Δ_t^p and Δ_t^w.

Wage Stickiness

- The competitive labour packer forms a composite labour service according to $H_t = \left(\int_0^1 H_t(j)^{(\mu-1)/\mu} dj \right)^{\mu/(\mu-1)}$ and sells onto the intermediate firm. where μ is the elasticity of substitution. For each j, the labour packer chooses $H_t(j)$ at a wage $W_{n,t}(j)$ to maximize H_t given total expenditure $\int_0^1 W_{n,t}(j) H_t(j) dj$.
- This results in a set of labour demand equations for each differentiated labour type j with wage $W_{n,t}(j)$ of the form

$$H_t(j) = \left(\frac{W_{n,t}(j)}{W_{n,t}} \right)^{-\mu} H_t^d \tag{9}$$

where $W_{n,t} = \left[\int_0^1 W_{n,t}(j)^{1-\mu} dj \right]^{\frac{1}{1-\mu}}$ is the aggregate wage index.
- H_t and $W_{n,t}$ are Dixit-Stigliz aggregators for the labour market corresponding to Y_t and P_t for the output market.

Wage Setting by the Trade-Union I

- Wage setting by the trade-union again follows the standard Calvo framework supplemented with indexation.
- At each period there is a probability $1 - \xi_w$ that the wage is set optimally. The optimal wage derives from maximizing discounted profits.
- For those trade-unions unable to reset, wages are indexed to last period's aggregate inflation, with wage indexation parameter γ_w.
- Then as for price contracts the wage rate trajectory with no re-optimization is given by $W^O_{n,t}(j)$, $W^O_{n,t}(j)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_w}$, $W^O_{n,t}(j)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_w}, \ldots$.
- The trade union then buys homogeneous labour at a nominal price $W_{h,t}$ and converts it into a differentiated labour service of type j.

Wage Setting by the Trade-Union II

- The trade union time t then chooses $W^O_{n,t}(j)$ to maximize real profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \frac{\Lambda_{t,t+k}}{P_{t+k}} H_{t+k}(j) \left[W^O_{n,t}(j) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} - W_{h,t+k} \right]$$

where using (9) with indexing $H^d_{t+k}(j)$ is given by

$$H_{t+k}(j) = \left(\frac{W^O_{n,t}(j)}{W_{n,t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} \right)^{-\mu} H^d_{t+k}$$

- Hence by analogy with (6) this leads to the optimal real wage

$$\frac{W^O_{n,t}}{P_t} = \frac{1}{(1-1/\mu)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} \left(\Pi^w_{t,t+k} \right)^{\zeta} H^d_{t+k} \frac{W_{h,t+k}}{P_{t+k}}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} \left(\Pi^w_{t,t+k} \right)^{\zeta} \left(\Pi^p_{t,t+k} \right)^{-1} H^d_{t+k}}$$

- Then by the law of large numbers:

$$W^{1-\mu}_{n,t} = \xi_w \left(W_{n,t-1} \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_w} \right)^{1-\mu} + (1-\xi_w)(W^0_{n,t}(j))^{1-\mu}$$

Wage Dynamics

- Now define

$$\Pi_t^p \equiv \frac{P_t}{P_{t-1}};\ \Pi_t^w \equiv \frac{W_{n,t}}{W_{n,t-1}};\ \tilde{\Pi}_t^w \equiv \frac{\Pi_t^w}{(\Pi_{t-1}^p)^{\gamma_w}};\ \tilde{\Pi}_t^p(\gamma) \equiv \frac{\Pi_t^p}{(\Pi_{t-1}^p)^{\gamma_p}}$$

- Then as for price dynamics we have

$$\frac{W_t^O}{W_{n,t}} = \frac{J_t^w}{W_t JJ_t^w}$$

$$JJ_t^w - \xi_w \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{(\tilde{\Pi}_{t,t+1}^w)^\mu}{\tilde{\Pi}_{t,t+1}^p(\gamma_w)} JJ_{t+1}^w \right] = H_t^d$$

$$J_t^w - \xi_w \mathbb{E}_t \left[\Lambda_{t,t+1} \tilde{\Pi}_{w,t+1}^\mu J_{t+1}^w \right] = -\frac{\mu}{\mu - 1} MRS_t MS_{w,t} H_t^d$$

where $MRS_t = -\frac{U_{H,t}}{U_{C,t}} = \frac{W_{h,t}}{P_t}$ and $H_t^d < H_t$ is aggregate labour used in production.

Output and Price Dispersion

- Wholesale output is now

$$Y_t^W = (A_t H_t^d)^{\alpha} K_{t-1}^{1-\alpha}$$

- Retail output and labour market clearing conditions must take into account price dispersion and wage dispersion.

- Integrating across retail firms, wholesale output market clearing gives:

$$\begin{aligned} Y_t^W &= \int_0^1 \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} dm\,(C_t + I_t + G_t) = \Delta_t^p Y_t \\ \Delta_t^p &= \xi_p \tilde{\Pi}_t^{\zeta} \Delta_{t-1}^p + (1-\xi_p)\left(\frac{P_t^O}{P_t}\right)^{-\zeta} \end{aligned}$$

Wage Dispersion

- Similarly integrating across differentiated labour types, labour market clearing gives:

$$\begin{aligned} H_t &= \int_0^1 H_t(j)dj = \int_0^1 \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\mu} dj\, H_t^d = \Delta_t^w H_t^d \\ \Delta_t^w &= \xi_w \tilde{\Pi}_{w,t}^{\mu} \Delta_{t-1}^w + (1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{-\mu} \end{aligned}$$

- Note that given labour supply H_t wage dispersion reduces labour employed H_t^d just as price dispersion reduces reial output.

Dynare Model Files in Folder NK

- **NK_SW.mod** with steady state files **NK_SW_steadystate.mod** which calls function **ss_fun_NK_SW**. No targeting of the Frisch elasticity and is closest to Smets and Wouters (2007)
- The model is **stationarized** in a **balanced growth steady state** - see Notes for full details
- There are **no flexi-price, flexi-wage blocs** in these codes and therefore they are only set up for an **implementable rule**.
- But see policy section with a flexi bloc that can have a **conventional Taylor rule**
- **graphs_irfs_compare_NK** Graph plotter for irfs

Exercises

1. Use the options in the code **NK_SW.mod** to explore the effect of introducing wage stickiness into the NK model. Compare impulse responses, volatility, co-movement and the contribution of shocks to the output variance for models with and without wage flexibility.
2. Do the same for external and internal habit.
3. Do the same for models with and without indexation.

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